

## Improvements in Network Modeling

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### Abstract

Maximal type of precedence relationships are the part of network scheduling theory since the beginning of the development of these techniques. Despite this fact current planning practice and widely known project planning applications solely support the use of minimal-type relationships. This restriction of the model usually does not result in a proper schedule. This paper would like to call the attention of the scheduling experts to the fact that "reinvention" and the use of maximal-type relationships can lead to a more adequate schedule. It also discusses some problems that can arise from the application of this type of logical dependencies and shows possible solutions.

### Keywords

Maximal type of precedence relationships, network modeling, Precedence Diagramming Method, time analysis, time-cost trade-offs

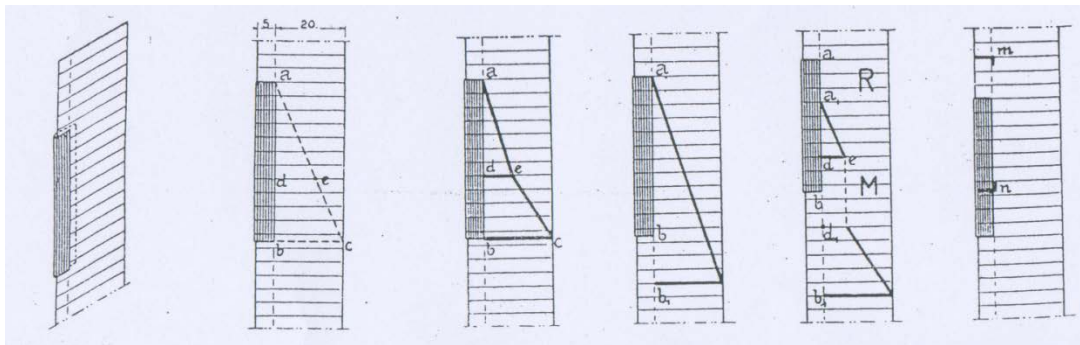
### 1. Historical Overview

Official history of network scheduling techniques dates back to the late fifties of the last century when the development of PERT, CPM and MPM techniques took place. However, an important forerunner has to be mentioned here, Adamiecki's harmonygraph also called as harmonogram (Adamiecki, 1931). According to him, the first version of the harmonogram was used in 1896, although we know only that version that was described and published in 1931. In the harmonogram each activity was represented by a scaled paper stripe, and the current schedule and duration of the activities were depicted by black clips. In the header of the stripes the name and the duration of the activity and the list of preceding activities could be seen. The stripes representing the preceding activities were always to the left of the stripe of the activity in question. A harmonogram can be seen below. (Figure 1)

time	From	-	-	-	A-1	B-1	...
	To	A-2	B-2,C	D-2	A-3	E-1	...
	activity	A-1<4>	B-1<4>	D-1<2>	A-2<4>	B-2<3>	...
1		█	█	█			
2							
3							
4		█	█				
5							
6					█	█	
7							
8							
9							
10							
11							
12							
13							
14							

Figure 1: Adamiecki's Harmonygraph (Weaver, 2006)

Admiecki has also developed a very sophisticated way to record the progression of the plan, (e.g. actual start and finish dates) and the historical data of tracking was displayed in the harmonogram in a very elegant way (Figure 2).



**Figure 2: Tracking Activities in the Harmonygraph (Adamiecki, 1931)**

All in all, the harmonogram could be mentioned as the first “modern” technique that has kept the logical dependencies among activities.

It is hard to tell whether the harmonogram has any impact on the modern network techniques (PERT, CPM, MPM). Probably directly not, because Adamiecki published his work in his mother tongue, that is in Polish. (Adamiecki, 1931) On the other hand, Adamiecki had a lot of presentations during his career and he could have inspired many practitioners, and academicians who used the result of his work in their own practice. In fact it is still not clear who could be credited for the invention of “traditional” time analysis, as PERT and CPM techniques were much more than this, and some forerunners of these techniques had to exist.

The history of PERT started in the mid-fifties with the Polaris program (Malcolm et al, 1958), when the Navy’s Special Projects Office (SPO) together with a consultant firm Booz-Allen and Hamilton and Lockheed were to develop a new technique to coordinate the above-mentioned development program. The result was a sophisticated stochastic network which has never gained so huge popularity as CPM or Precedence Diagramming. We have to mention here that some kind of reinvention of the PERT concept comes back in today’s Precedence Diagramming-based applications in order to perform risk analysis. The history of CPM also dates back to the late fifties, when Du’Pont, an international chemical concern developed a new method to help and optimize its maintenance work. The team, which was led by Kelley and Walker, developed a technique that could have been used to optimize the project direct cost. (Kelley and Walker, 1959a) (Kelley and Walker, 1961b) (Kelley and Walker, 1989b) The computer they used for the calculation was a Univac, the third in the world but the first in the civil, non-military world. More on the history of CPM can be read in the work of Weaver (Weaver, 2006). This model was a cost optimization model and the algorithm was based on the relatively newly invented linear programming (Dantzig, 1949).

The model was too sophisticated for engineers; computers and computer applications were not available. Therefore engineers made simplifications and instead of performing cost optimization, they performed only time analysis, which could be done manually as well. During the decades hundreds of embellishments, new algorithms have been developed but cost optimization is still not part of the project management practice, and CPM has less and less weight in practical applications due to the relatively difficult graphical portraying and its narrow modeling capabilities.

Parallel to the developments taken place in the United States, Europe invented its own network technique the so-called Metra Potential Method (MPM) (Roy, 1959), (Roy, 1962). This method applied the activity-

on-node concept in displaying the network, and laid down the basis of applying the different precedence relationships. In fact it was so sophisticated that it introduced the maximal relationships! Despite the fact that Roy published his work in French, European researchers discovered his work and added new precedence relationships to the model. At the early mid-sixties MPM could handle all the minimal and maximal relationships which are used today. Minimal relationships are: Start-to-Start, Finish-to-Start, Finish-to-Finish and Start-to-Finish relations with lead/lag time, while maximal relationships are known as: max-Start-to-Start, max-Finish-to-Start, max-Finish-to-Finish and max-Start-to-Finish relations with lead/lag time.

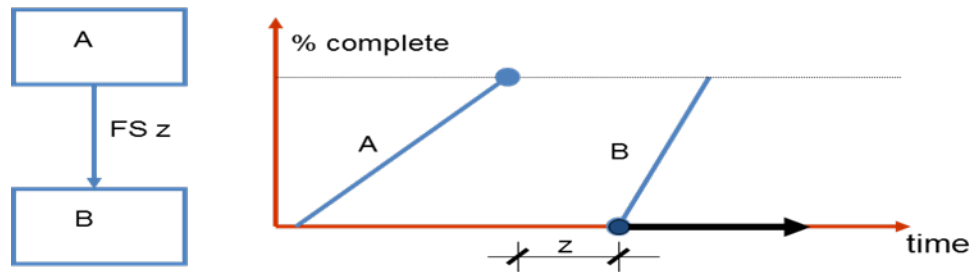
Almost at the same time, or a bit after the work taken place in Europe, the Precedence Diagramming Method was developed in the United States based on the work of different researchers e.g. John Fondahl, who introduced the activity-on-node concept for CPM (Fondahl, 1962). Precedence Diagramming and MPM network technique could be seen as the same, they are both an activity-on-node network, where different minimal and maximal logical relationships between the start or finish point of any two activities can be given, and the primary aim of its analysis is to determine the project duration and the early and late start and finish dates of the activities. Maximal relationships were not allowed in the original Precedence Diagramming, and widely known scheduling programs developed in the US still cannot handle maximal relationships, while “locally” used European applications can do this. This is the reason why maximal relationships are still not an essential part of the project management practice. For the last decade maximal relationships have been reinvented, and have been subject of different research and investigation.

## 2. Precedence Diagramming

Precedence Diagramming – or MPM (as it called in Europe) – is a deterministic network technique with the primary goal of time analysis. It has two important elements: activities and precedence relations. Definitions for activities and precedence relations are the following.

*Definition 1. (Activity): An activity is a process that should be carried out with the same intensity without interruption.*

*Definition 2. (Minimal relationships): A minimal relationship describes the minimum duration ( $z$ ) that has to elapse between the start or finish points of any two activities. The following minimal precedence relations can be used: Start-to-Start- $z$ ; Finish-to-Start- $z$ ; Start-to-Finish- $z$ ; Finish-to-Finish- $z$ .*



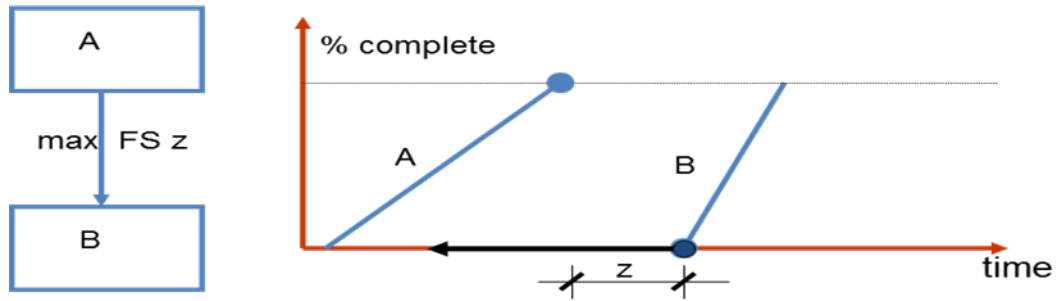
**Figure 3: PDM and Linear Scheduling Representation of FS $z$  relationship**

Figure 3 explains the definition in case of using Finish-to-Start- $z$  relationship. The half-line that originates from the start of B and leads to the right shows the possible starts for activity B.

*Definition 3. (Maximal relationships): A maximal relationship describes the maximum duration ( $z$ ) that is allowed to elapse between the start or finish points of any two activities. The following maximal*

precedence relations can be used: max-Start-to-Start-z; max-Finish-to-Start-z; max-Start-to-Finish-z; max-Finish-to-Finish-z.

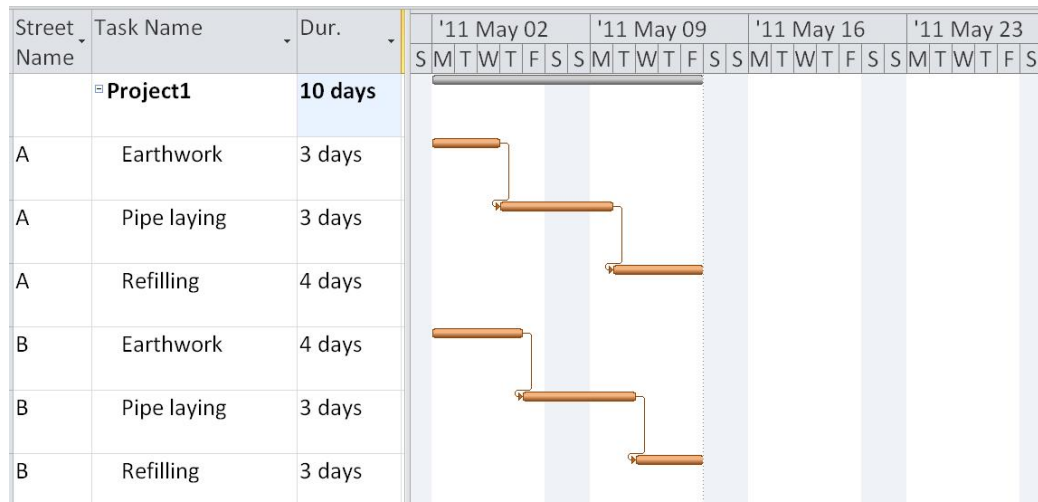
The figure below explains the definition in case of max-Finish-to-Start-z relationship. The half-line that originates from the start of B and leads to the left shows the possible starts for activity B.



**Figure 4: PDM and Linear Scheduling Representation of max-FS<sub>z</sub> relationship**

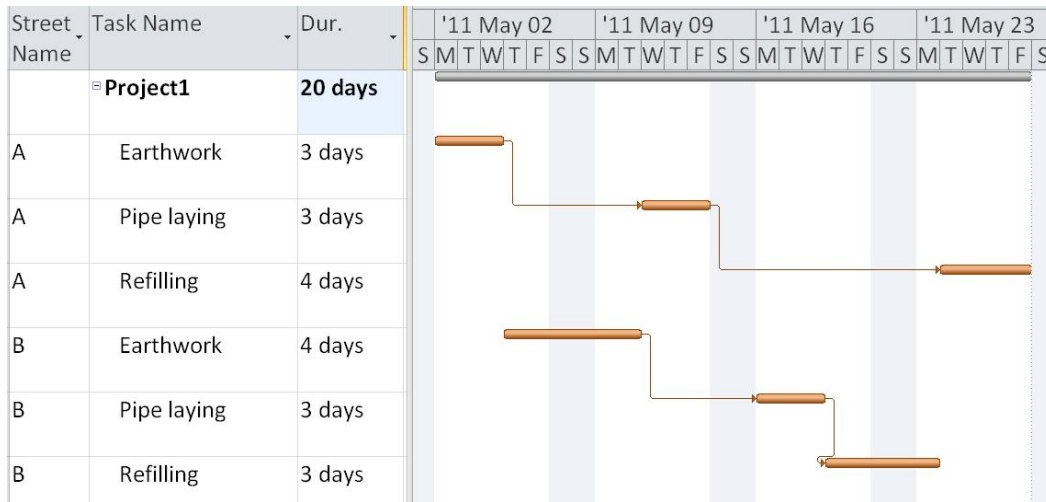
For justifying the usefulness of maximal relations the following examples will be shown.

A small sewage pipeline project has to be carried out in two parallel streets: A and B. Earthwork, pipe laying, and refilling are consecutive tasks, so the relation between them is FS<sub>0</sub> in both streets. This can be seen on the figure below. (Figure 5)



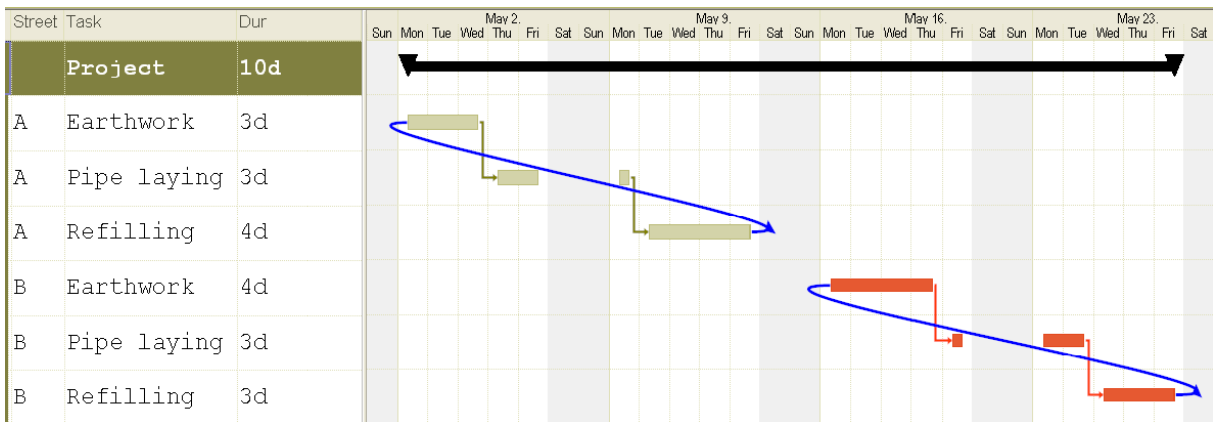
**Figure 5: Example #1. No Leveling Performed**

The same group of workers carry out all the activities, so because of limited resources, leveling has to be performed, the result of which can be seen on Figure 6.



**Figure 6: Example #1. Results after Leveling**

It can be seen that the duration of the project has changed from 10 to 20 days, and the length of work in both streets has increased: from 10 to 20 days in street A, and from 10 to 13 days in street B. Because of the resource limit it is obvious that the minimal project duration cannot be shorter than 20 days, but a definitely better solution would be, if the length of work could be the minimum in both streets, eg. 10 days in A and B as well. This is shown on Figure 7, where maximal types of precedence relations were applied to determine the maximum duration of the work in street A and street B as well. Maximal relations (maxFS10 and maxFS10) are plotted in curved lines.

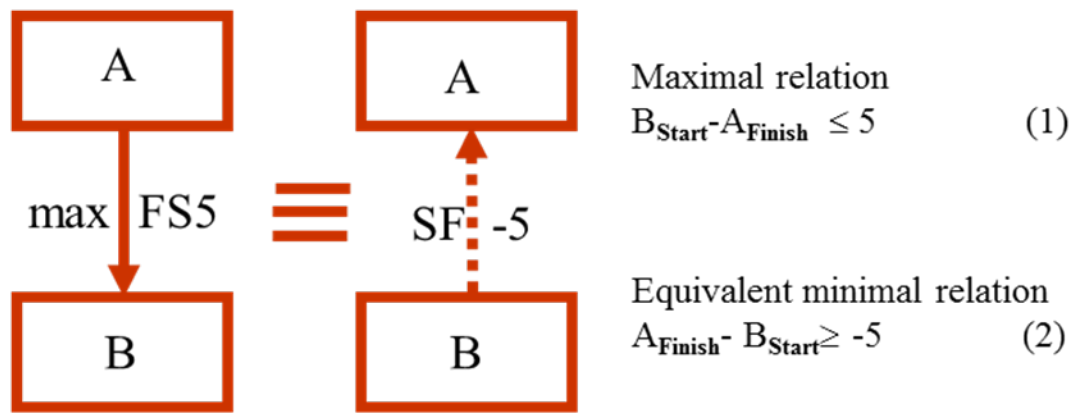


**Figure 7: Applications of Maximal Relations for Getting a More Realistic Result after Leveling**

Tremendous examples could be cited to justify the usefulness of maximal precedence relations, but as the size of this paper is limited, they will not be.

### 3. Time Analysis and Resource Planning When Maximal Relationships Are Applied

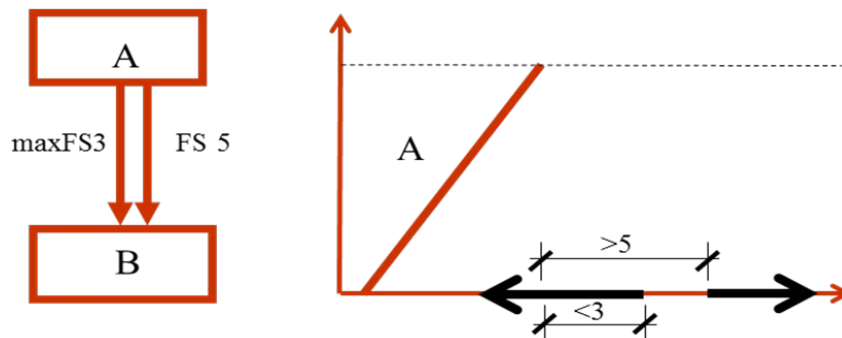
When maximal relations are allowed some problems can arise during time analysis. Traditional time analysis cannot be applied in these cases. The reason for this is that during the calculation maximal relations are converted into minimal ones. As a result of the conversion, the direction of the relations will be the opposite. This is explained in Figure 8.



**Figure 8: Max FS z and its Equivalent Minimal Relation**

To justify this, (2) should be simply multiplied by -1. During the conversion loops can appear in the network. Algorithms that can solve time analysis are based on longest path algorithms that can handle loops. Algorithms used in case of maximal relations are based on iteration where at least one activity can get its final start and finish dates in each and every iteration. This means that maximum  $n$  iterations have to be performed for getting the final result. This can be read about for example in Hajdu's book (Hajdu, 1997). (Let  $n$  be the number of activities in the network.)

Maximal relations can produce situations when no solution exists. Figure 9 will prove this statement. It can be seen that the two half-lines representing the possible starts of B, based on the relations, have no common parts. Time analysis in this case would lead to a never-ending chain of iterations, so if we do not have the final results within  $n$  iterations, then we can be sure that no solution exists.



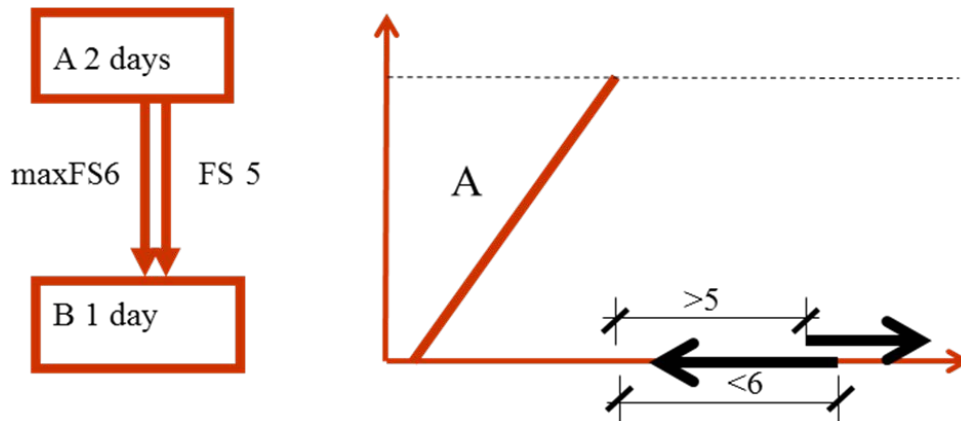
**Figure 9: Scheduling Conflict Can Arise When Maximal Relations Are Used**

Leveling algorithms that are based on the order of activities cannot be used during time analysis either. As widely spread project management applications are based on this approach, it is not enough to change the algorithms used for time analysis, leveling algorithms have to be renewed as well.

#### 4. Calendars in Scheduling

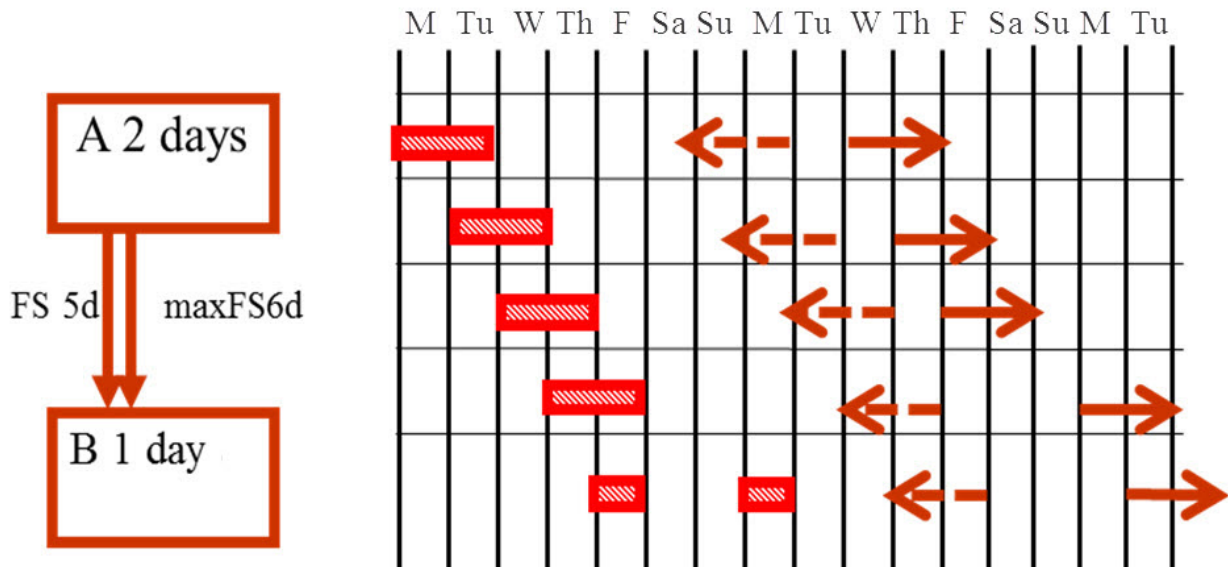
Calendars are, unfortunately, very rarely subjects of detailed investigations. Time analysis or different resource allocation algorithms are usually not tested when calendars are used in projects, despite their serious effects on calculations. This is especially true when both minimal and maximal types of relations are used in the network. It can be stated and proved that sometimes no solution exists when we would say - based on calculation without calendars - that there is, and there is a solution when we would say that no solution exists. The first statement will be proved below. Let's examine the following two figures. (Figure 10 and 11)

In Figure 10 the two half-lines stand for the possible starts of B defined by the two relations. As they have a common part, a solution for this network exists.



**Figure 10: Solution Exists**

Let's use different calendars for the same project. Let activity A and B and FS5 relation use a 5-day calendar (working days are from Monday till Friday, weekends are non-working periods) while maxFS6 uses a 7-day calendar, meaning that every day is a working day. Figure 11 shows all the possible starts of activity A within a week. Arrows plotted with continuous lines show the possible starts of B based on the FS5 relation, while arrows plotted with dashed lines show the possible starts of B based on the maxFS6 relation. While a solution existed when there were no calendars in the project (it practically means that only one calendar is allowed), in case of more calendars there is no solution for this project.



**Figure 11: Solution Does Not Exist in Case of Different Calendars**

## 5. Conclusions

Maximal relations are an important part of project scheduling theory, despite this only a few applications can handle these kinds of relations. Precedence Diagramming and MPM techniques developed concurrently, their basis are the same, that way maximal relations could easily be incorporated into Precedence Diagramming Method.

Maximal type of relations requires different calculation rules during time analysis, and resource leveling as well. Usage of different calendars in a project can lead to serious computational problems, previously solvable projects become non-solvable and vice-versa.

Standardization of the calculation, when maximal relations are allowed, – especially in case of different activity, resource and relationship calendar – would be essential. In the absence of this standardization, different tools will be applied, which will result in different solutions for the same project, due to their own calculation rules. Such situations should be avoided.

This standardization would be led by the leading project management organizations.

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