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Solution of Unbalanced Transportation Problems Using Simplified Generalized Network Algorithm (SGNA)

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Abstract

This paper deals with an application of Generalized Networks on unbalanced transportation problems in construction site management. Sample problem is about minimal cost-maximal flows of concrete materials required from construction site. Two factories can supply any of three construction site with a particular product (concrete materials). The demand for this product from each of the site has different capacities. This problem can be called unbalanced transportation problem. Because, demand amount and supply amount are not equal to each other. In this problem, supply center (concrete plants) can not supply the demand. The capacities of supply center are less than demand center (construction sites). Maximum production at concrete plants, the variable production cost per m^3, and the transportation cost per m^3 from each factory to each site are different.

In this paper, a new algorithm called Simplified Generalized Network Algorithm (SGNA) based on networks has been derived from Generalized Networks (GN) for solving sample problem. The classical solutions of the sample unbalanced transportation and SGNA solutions of same unbalanced transportation problem have obtained using analytical solution techniques. These solutions have been classified and compared with each other. The suggested new algorithm SGNA give us same solutions of other solution algorithms. This shows that SGNA is capable and useable for solving of unbalanced transportation problems mentioned construction site management.

Keywords

Unbalanced transportation problem, Generalized network

1. Introduction

The unbalanced transportation problems play an important role in material logistics and supply chain management for reducing cost and improving service in construction site management. Therefore, the goal of material management and material transportation is to find the most cost effective way to transport the goods. The cost of transportation consists of freightage and inventory costs. The freightage is about of 35 % of logistic cost of construction companies. It has a big percentage in total project cost especially in concrete plants, construction of soil-dams, construction of steel structures and construction of highways. In the literature, we can find a lot of paper about transportation problems and its applications.

In Chau's paper, a two-stage dynamic model is developed to assist construction planners to formulate the optimal strategy for establishing potential intermediate transfer centers for site-level facilities such as batch plants, lay-down yards, receiving warehouses, various workshops, etc. Under this approach, the solution of the problem is split into two stages, namely, a lower-level stage and an upper-level stage. The former can be solved by a standard linear programming method, whereas the latter is solved by a genetic algorithm. The efficiency of the proposed algorithm is demonstrated through case examples (Chau, 2004).

The capacitated transportation problems with bounds on RIM conditions have been solved by Dahiya and Verma (2006); then the total cost bounds of the transportation problem with varying demand and supply have been solved by Shiang-Tai Liu. In real world applications, the supply and demand quantities in the transportation problem are sometimes hardly specified precisely because of changing economic conditions. Liu has investigated the transportation problem when the demand and supply quantities are varying (Liu, 2003). A multi-parametric demand transportation problem has been solved with a proposed algorithm known as network and parametric algorithms (Filippi and Romanin-Jacur, 2002).

In many distribution problems, the transportation cost consists of a fixed cost, independent of the amount transported and a variable cost, proportional to the amount shipped. Adlakha and Kowalski have developed a quick sufficient condition to identify candidate markets and supply points to ship more for less in fixed-charge transportation problems (Adlakha and Kowalski, 1999). Sharma and Prasad have given a new procedure to solve the dual of the well-known uncapacitated transportation problem. They have given a heuristic that obtains a very good starting solution (with a duality gap of less than 2%) for the primal transportation problem (Sharma and Prasad, 2002).

In this paper, we have suggested a new algorithm based on Generalized Networks (GN) usable for solving the unbalanced transportation problems that is called Simplified Generalized Network Algorithm (SGNA). The minimal cost-maximal flow solution of unbalanced transportation problem and SGNA solution of the same problem have obtained using graphical techniques. These solutions have been classified and compared with each other. The capability and usage of the new suggested graphical method, SGNA, has been explained by using sample unbalanced transportation problem that may be occurred in construction site.

2. General Description of Unbalanced Transportation Problems

The basic elements required to describe and formulate an unbalanced transportation problems can be classified by using three parameters. These are unit cost of transportation, the availability of the production centers, and the requirements of demand centers. The supply amount and demand amount are not equal to each other in unbalanced transportation problems. The general formulation of the unbalanced transportation problem can be derived as shown in Equation 1 and 2. The graphical structure of the model to be formulated can be prepared as shown in Figure 1.

Usually, the input information required for the formulation of the model can be displayed in a rectangular array which contains the cost parameters for all possible routes, and the availability and demand figures for the supply and demand centers, respectively. As an illustration, such an arrangement can be shown in Table 1 according to Figure 1.

$$Min\ Z = \sum_{i=1}^{i=m} \sum_{j=1}^{j=n} C_{ij} . X_{ij} \qquad X_{ij} \geq 0\ (i = 1,2,...,m; j = 1,2,...,n) \tag{1}$$

$$subjected\ to \sum_{j=1}^{j=n} X_{ij} \leq S^{i}\ \ (i = 1,2,...,m) \quad subjected\ to \sum_{i=1}^{i=m} X_{ij} \geq D^{j}\ \ (j = 1,2,...,n)$$

$$\sum_{i=1}^{i=m} S^{i} \geq \sum_{j=1}^{j=n} D^{j} \tag{2}$$

Where C_{ij} can be called the cost of transportation and Xij is the capacity of transportation.

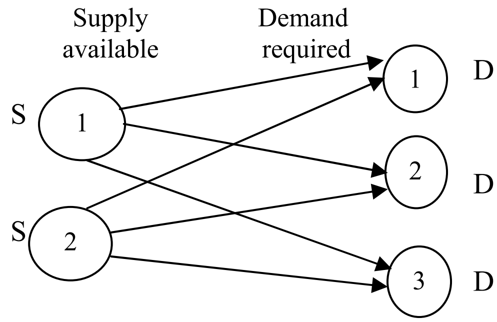

Figure 1: Graphical Structure of the Transportation Problem

Table 1: Input Information for the Transportation Problem

Supply Center	Demand Center			
	D_1	D_2	D_3	Total Demand
S_1	C_{11}, X_{11}	C_{12}, X_{12}	C_{13}, X_{13}	S^1
S_2	C_{21}, X_{21}	C_{22}, X_{22}	C_{23}, X_{23}	S^2
Total Supply	D^1	D^2	D^3	Supply/Demand

3. Derivation of Simplified Generalized Network Algorithm

The flows that are transmitted across the arcs of a generalized network can be modified by using four parameters (Aytekin, 2001). Considering a directed network of arc *(I, J)*, F_{ij} is a flow passing from node *I* to node *J* at a unit cost C_{ij}. The allowable amount of flow along *(I, J),* must be at least equal to L_{ij} and at most equal to U_{ij}. Additionally, the arc multiplier can be represented by the symbol A_{ij}. The arc parameters can be denoted graphically as shown in Figure 2.

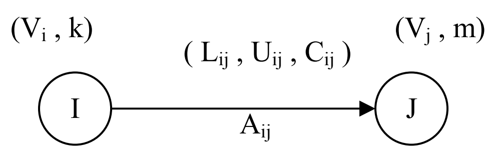

Figure 2: Arc Parameters of a Simple Generalized Network

3.1 Phase I: The Minimum Cost Chain

To find the minimum cost chain of a generalized network it is necessary to define a new variable, V_i for each node *I*, in the network. New variable can be referred to as a *node potential*. Starting with the potential of first node of network, potential should be zero ($V_i=0$), and potential of all other nodes should be infinity ($V_j=\infty$) (Cohen and Megiddo, 1997). The potential of a node can be interpreted as a cost of supplying 1 unit of flow from that node. In a directed chain, this is the lowest cost of transporting 1 unit of flow from source node to node under consideration. The potential of a node can be controlled by the costs and arc multiplier of the chain that delivers the flow to that node. If the transporting cost per unit of flow to reach a node *I* is V_i, the cost per unit of flow to reach another node *J*, through arc *(I, J)* can be obtained by using Equation 3 (Aytekin *et al.*, 2008).

$$\overline{V_{ij}} = \frac{V_i + C_{ij}}{A_{ij}} \tag{3}$$

Where C_{ij} is the cost of transmitting 1 unit of flow from node *I* to *J*, and A_{ij} is the multiplier of arc *(I, J)*.

After applying Equation 3 to all arcs terminating at node *J*, the potential of this node can be obtained by Equation 4.

$$\mathrm{V_j} = \min_{i \in B_j} \overline{V_{ij}} = \min_{i \in B_j} \left[\frac{V_i + C_{ij}}{A_{ij}} \right] \tag{4}$$

In Equation 4, B_j is the set of nodes directly connected to node *J* by arcs with oriented from *I* node to *J* node. All arcs terminating at node *J* can bring same potentials. In this situation, we should compare the capacities of arcs. If all arcs terminating at node *J* have same potentials the arc having biggest capacity should be chosen. This process is continued until all arcs are scanned (Aytekin *et al.*, 2008).

3.2 Phase II: Constructing the Reduced Network

Once flow is transmitted into network through phase I, the residual capacities of unsaturated arcs can be obtained by subtracting the current flows from original arc capacities. These residual capacities can be adjusted by modifying the flows in the original network. To perform such adjustments the original network can be converted into a network referred to as a reduced network using Table 2.

Table 2: Adjustments of Arc Parameters for the Reduced Network

Before transmitting flow	After transmitting flow	Constraint
C_{ij}	$^*C_{ij} = C_{ij}$	if $F_{ij} < U_{ij}$
U_{ij}	$^*U_{ij} = U_{ij} - F_{ij}$	if $F_{ij} < 0$
A_{ij}	$^*A_{ij} = A_{ij}$	if $F_{ij} < U_{ij}$

3.3 Phase III: Routing the Maximal Flow Chain and Executing the Flow-Augmentation

To route the maximal flow chain, we must start the final node of networks. Let last node of network has a potential *V*j that can be obtained from phase I, and source node *m* that is node receiving the flow using this potential (Jensen and Barnes, 1980). The last node with having potential can be illustrated (*Vi*, *m)*. In this illustration, *m* shows us which node is transporting the flow to this node. Using this illustration we can get the maximal chain and its arcs. These chain and arcs can be denoted by using Equation 5.

$$\begin{aligned} N_c &= \{m, k, p, n, \ldots\ldots \quad m \neq k \neq p \neq n\} \\ A_c &= \{(i, j), (j, n), (n, p), \ldots\ldots\ldots\ldots, (k, m)\} \end{aligned} \tag{5}$$

After obtaining the maximal chain and arcs, adjusting arcs parameter for the reduced network can be obtained using Table 2. Finally, these three phases should be repeated until no flow transmitting from source (first node) to demand (last node) of the network. This repetition is equal or less than number of supply multiplied by number of demand times. The maximal flow of the chain can be obtained by using

Equation 6 and the cost of flow also can be obtained by multiplying F_k and potential of last node (Aytekin *et al.*, 2008).

$$F_k = \min_{k \in A_c} \frac{U_k}{\prod_{ar}} \tag{6}$$

4. Definition of Sample Unbalanced Material Transportation Problem and SGNA Solution

The basic elements required to formulate and solve an unbalanced transportation problems can be denoted using the unit cost of transportation, capacity of supply centers and requirements of demand centers. For this sample unbalanced transportation problem, we have considered two concrete plants (supply centers) and three construction sites (demand centers). Firstly, graphical structure of the sample problem must be done as shown in Figure 1. Using this graphical structure, sample network of unbalanced transportation problem can be modified for solving SGNA. This sample SGNA network of unbalanced transportation problem has been shown in Figure 3.

In this situation, two concrete plants have different production capacity and supply capacity for three construction site. The capacities of concrete plants (supply centers), the concrete supplied from concrete plants to construction sites, and daily requirements of each construction site have been shown in Table 3.

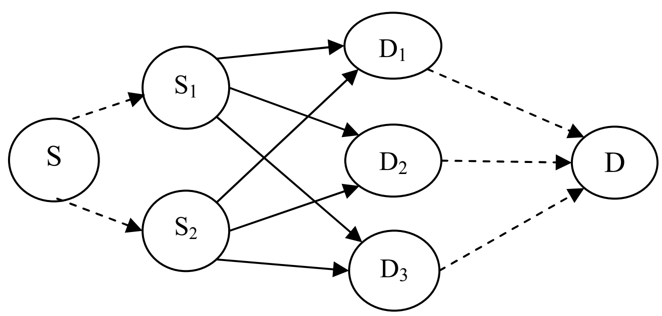

Figure 3: Sample Network of Unbalanced Material Transportation Problem for SGNA Solution

Table 3: The Properties of Sample Transportation Problem

Concrete Plants Capacity (m^3/day)	**Concrete Plants**	**Construction Sites**			**Construction Sites**	**Site Requirement**
		D_1	**D_2**	**D_3**		**Demand (m^3/day)**
		D/Cp/C	**D/Cp/C**	**D/Cp/C**	**D_1**	300
500	**S_1**	45/135/3.00	50/125/3.50	43/160/2.50	**D_2**	270
450	**S_2**	55/155/3.50	40/135/2.50	52/140/3.50	**D_3**	310

D: Distance (km)   Cp: Transportation Capacity (m^3/day)   C: Cost of transportation (\$/m^3)

The capacities of concrete plants, the amount of daily concrete supplied from concrete plants to construction sites and daily requirements of construction sites have been verified to convert the SGNA network parameters. Assignments of SGNA network parameters can be confirmed as shown in Table 4. In this table, multiplier parameters and lower bound of capacities of all arcs have been supposed having zero value. Because lower bound of all arcs should be zero according to the assumptions that mentioned above.

Table 4: Assignments of SGNA Network Parameters of Sample Unbalanced Material Transportation Problem

Arc	L_{ij}	U_{ij}	C_{ij}	A_{ij}	Arc	L_{ij}	U_{ij}	C_{ij}	A_{ij}
S-S_1	0	500	0	1.00	**S_2-D_2**	0	135	2.50	1.00
S-S_2	0	450	0	1.00	**S_2-D_3**	0	140	3.50	1.00
S_1-D_1	0	135	3.00	1.00	**D_1-D**	0	300	0	1.00
S_1-D_2	0	125	3.50	1.00	**D_2-D**	0	270	0	1.00
S_1-D_3	0	160	2.50	1.00	**D_3-D**	0	310	0	1.00
S_2-D_1	0	155	3.50	1.00					

SGNA solution of material transportation problem can be obtained after less than 7 iterations of phase I, II and II that mentioned above. Applying the phase I, II and III for first iteration have been shown in Table 5.

After finishing the first iteration, the reduced SGNA network must be performed all iterations. Final results of SGNA solution of sample unbalanced transportation problem can be arranged as shown in Table 6.

Table 5: Application of Phase I, II and III for the First Iteration

Node	Arcs	V_i	C_{ij}	A_{ij}	$\overline{V}_{ij}$	V_j	Notation
S_1	S-S_1					0	(0-S)
S_2	S-S_2					0	(0-S)
D_1	S_1-D_1	0	3.00	1.00	3.00	3.00	(3.00-S_1)
	S_2-D_1	0	3.50	1.00	3.50		
D_2	S_1-D_2	0	3.50	1.00	3.50	2.50	(2.50-S_2)
	S_2-D_2	0	2.50	1.00	2.50		
D_3	S_1-D_3	0	2.50	1.00	2.50	2.50	(2.50-S_1)
	S_2-D_3	0	3.50	1.00	3.50		
D	D_1-D	3.00	0	1.00	3.00	2.50	(2.50-D_2)
	D_2-D	2.50	0	1.00	2.50		
	D_3-D	2.50	0	1.00	2.50		

Arcs	k	U_k	$\prod_{ar}$	F_k
S-S_2	1	450	1	450
S_2-D_2	2	135	1	135
D_2-D	3	270	1	270

Unit cost for first iteration: 2.50 \$/m^3 **Flow:** 135 m^3/day

Concrete transported from S_2 to D_2 is 135 m^3/day **Total Cost:** 135*2.50=337.50 \$

Supply Center	Supply	D1	D2	D3	Cost
S_1	500				
S_2	450		135		337.50

To compare SGNA solution and classical solution of the sample problem, we have chosen minimal cost-maximal flow problem solution method. This method is based on graphical network model (Glover *et al*., 1992). The used parameters and solution of the sample unbalanced transportation problem using this method can be prepared as shown in Table 7 and Table 8 respectively.

Table 6: Final Results of SGNA Solution of Sample Unbalanced Transportation Problem

Capacity	Concrete Plants	D_1	D_2	D_3	Cost	Total Cost
500	**S_1**	135	125	160	1242.50	
450	**S_2**	155	135	140	1370.00	
950	**Supplied**	**290**	**260**	**300**	**850**	**2612.50**

Table 7: The used Parameters of the Sample Unbalanced Transportation Problem to Solve as a Minimal Cost-maximal Flow Problem

Start Node	Finish Node	Capacity	Cost	Start Node	Finish Node	Capacity	Cost
S	S_1	500	0.00	S_2	D_2	135	2.50
S	S_2	450	0.00	S_2	D_3	140	3.50
S_1	D_1	135	3.00	D_1	D	300	0.00
S_1	D_2	125	3.50	D_2	D	270	0.00
S_1	D_3	160	2.50	D_3	D	310	0.00
S_2	D_1	155	3.50				

Table 8: Solution of the Sample Unbalanced Transportation Problem as a Minimal Cost-maximal Flow Problem

N^{th} Paths	Minimal Cost	Routes	Maximal Flow for N^{th} Shortest Path	Cost of Flows
1^{st} Shortest Path	2.50	S-S_1-D_3-D	160	400.00
2^{nd} Shortest Path	2.50	S-S_2-D_2-D	135	337.50
3^{rd} Shortest Path	3.00	S-S_1-D_1-D	135	405.00
4^{th} Shortest Path	3.50	S-S_2-D_1-D	155	542.50
5^{th} Shortest Path	3.50	S-S_2-D_3-D	140	490.00
6^{th} Shortest Path	3.50	S-S_1-D_2-D	125	437.50
		Total	**850**	**2612.50**

5. Results and Discussion

Construction site management is generally highly dynamic and complex by their very nature and activities. Hence, it is highly desirable to be able to formulate the optimal strategy for transportation facilities at different times of the project. The principal objective is to minimize the total cost, which comprises the transportation types, handling, capital, and operating costs at potential intermediate transfer centers of various plant and material resources over the entire project duration. Because of this, unbalanced transportation problems play an important role in material logistics and supply chain management for reducing cost and improving service in construction site management.

Based on Table 9, it can be seen that the total cost of classical minimal cost-maximal flow problem solution is equals to the total cost of SGNA solution. Both of classical and SGNA solutions have been calculated after 6^{th} iteration. The classical solution technique can give us a solution according to a rule that network structure of unbalanced transportation problem must be arranged as a minimal cost-maximal flow problem. SGNA solution technique is capable to find solutions using only one network that can be easily arranged mentioned above. According to the sample unbalanced transportation problem, it can be seen that the requirements of all demand centers have not been supplied from only one supply center. For example, D_3 construction site requires 310 m^3 per day concrete but S_1 concrete plant can supply 135 m^3

per day concrete. How can we get the remaining amount or where can we get the remaining amount from? In this situation, SGNA method gives us the optimal solution for sample unbalanced transportation problem.

Table 9: The Results of the Classical and SGNA Solution for the Sample Unbalanced Transportation Problem

Results of SGNA Solution Technique						Results of Classical Solution Technique					
Supply Center	Capacity (m^3)	Demand Center			Cost ($)	Supply Center	Capacity (m^3)	Demand Center			Cost ($)
		D_1	D_2	D_3				D_1	D_2	D_3	
S_1	500	135	125	160	1242.50	S_1	500	135	125	160	1242.50
S_2	450	155	135	140	1370.00	S_2	450	155	135	140	1370.00
Total Cost					**2612.50**	Total Cost					**2612.50**
Total Capacity of Supply Centers					950	Total Capacity of Supply Centers					950
Total Capacity of Demand Centers					850	Total Capacity of Demand Centers					850

We have seen that the suggested new graphical solution technique SGNA is capable to solve the unbalanced transportation problems in a short time after a few iterations less than classical techniques. In addition, SGNA solution technique gives desirable, real solutions especially in not equal supply demand transportation problems and transportation from different supply centers in different amounts.

6. References

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