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Application of the Monte-Carlo Method to Determine the Costs for Building Projects – Influence of Ranges on Probability Distribution

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Abstract

This paper sets out to demonstrate the calculation of construction costs whilst considering key construction management parameters. Beyond a simple, deterministic method, other options for calculation are shown that rely on probability calculus. The approaches described to determine construction costs are illustrated by a building project example. The Monte-Carlo method will then be applied on the basis of the deterministic calculation mode used initially. The Monte-Carlo calculation sequence will be both described and displayed graphically. For each simulation exercise, distribution functions will be selected for the input parameters, and minimum and maximum values will be determined, as well as most likely values, as far as reasonably possible. The @RISK software will be used to carry out the simulations. The results will be shown as probability distributions for the output parameters defined in a preceding step. The results of several simulations with varying distribution functions will be compared to each other and analyzed. The comparison should show whether it is useful to apply weighted triangles as distribution functions. The influence of a reduction in the input parameter range on the probability distributions will also be determined. Such a range reduction is enabled by a more detailed knowledge of the prevailing site, management, structural and process conditions. The degree of uncertainty and fuzziness can be lowered as a result. The paper is to demonstrate and analyze the influence of the input parameter improvement on the results.

Keywords

Construction time, construction costs, @RISK, Monte-Carlo simulation, building projects, construction works, probability distribution, range reduction

1. Introduction

In the construction industry, the job cost system is normally used to determine costs and, ultimately, prices. The costs of production comprise the costs of individual work items and site overheads. Item costs are sub-divided into labour, equipment and materials costs. For the purpose of preliminary costing, such as in early project phases, to arrive at initial cost estimates, or to check previously performed calculations, reference values are taken from comparable projects and/or from the literature. The uncertainties of these amounts are greater than during the more thorough cost considerations at the detailed costing stage. Using both an optimistic and a pessimistic scenario, a range of production costs can be determined. The intended production cost should then lie within this range. Depending on the input parameters selected for the calculation, the spread between the subjective extremes gets smaller or greater. Deterministic calculations each lead to individual results, which are then added up to a total in a non-systematic manner. In order to systematically account for ranges, the Monte-Carlo method is applied using the @RISK software. This approach makes it possible to determine the probability distributions for the relevant output parameters.

For the purpose of the investigations outlined in this paper, the production costs of reinforced concrete works are determined for the construction of the shell of a performing arts centre building.

2. Fundamentals of output computation

For the purpose of the preliminary calculation of the project production cost, the individual deterministic computation steps are shown. To determine labour cost, the total labour consumption rate of the reinforced concrete works is calculated and multiplied by the concrete quantity. The resulting hourly wage total is multiplied by the mean wage costs, which gives the final labour cost. Unit costs are determined for equipment and materials and subsequently multiplied by the concrete quantity. The addition of labour, equipment and materials costs results in the individual item costs. Site overheads are included by adding a mark-up to the item costs. The production costs considered in this paper are equivalent to the total of item costs and overheads.

2.1 Item costs for reinforced concrete works – labour cost

For reinforced concrete works, the total labour consumption rate TCR_{RCW} [hr/m³], which includes formwork, reinforcement and concrete placement works, is calculated using Eq. 1. The amount of the total labour consumption rate shows the degree of labour intensity of the reinforced concrete works within the project considered.

$$TCR_{RCW} = CR_{A,FW} * FR_{A,BD} + CR_{A,RW} * RR_{A,BD} + CR_{A,CW} \tag{1}$$

The first term is the product of the average formwork placement labour consumption rate $CR_{A,FW}$ [wh/m²] and the formwork ratio $FR_{A,BD}$ [m²/m³]; the second is the product of the average reinforcement work labour consumption rate $CR_{A,RW}$ [wh/t] and the reinforcement ratio $RR_{A,BD}$ [t/m³]; the last term represents the average labour consumption rate of concrete placement $CR_{A,CW}$ [wh/m³]. Eq. 1 is used to either estimate or precisely calculate the mean values. A more accurate calculation is carried out as part of a detailed analysis. Depending on the item considered, the total labour consumption rate can be calculated either for the entire structure or for individual groups of structural components. The accuracy of the results usually increases in line with the degree of detail of the analysis. The item costs for the labour component $DC_{LC,RCW}$ [€] are equivalent to the product of the total labour consumption rate TCR_{RCW} [wh/m³], the mean wage costs WA_{RCW} [€/wh] and the concrete colume Q_c [m³], as shown in Eq. 2.

$$DC_{LC,RCW} = TCR_{RCW} * WA_{RCW} * Q_c \tag{2}$$

2.2 Item costs for reinforced concrete works – costs of equipment and materials

Equipment and materials costs per unit (unit costs) $c_{A,EM,RCW}$ [€/m³] for reinforced concrete works are related to the concrete quantity and calculated using Eq. 3.

$$c_{A,EM,RCW} = c_{A,EM,FW} * FR_{A,BD} + c_{A,EM,RW} * RR_{A,BD} + c_{A,EM,CW} \tag{3}$$

For the formwork placement works [€/m³], unit costs are calculated as the product of the formwork unit costs $c_{A,EM,FW}$ [€/m²] and the formwork ratio $FR_{A,BD}$ [m²/m³]. Multiplying the reinforcement unit costs $c_{A,EM,RW}$ [€/t] by the reinforcement ratio $RR_{A,BD}$ [t/m³] results in the unit costs of the reinforcement expressed by the second term of the equation. The last term represents the unit costs of concrete $c_{A,EM,CW}$

[€/m³]. To calculate the item costs of equipment and materials $DC_{EM,RCW}$ [€], the unit costs $c_{A,EM,RCW}$ [€/m³] calculated using Eq. 3 in the preceding step are multiplied by the concrete quantity Q_c [m³] (see Eq. 4).

$$DC_{EM,RCW} = c_{A,EM,RCW} * Q_c \tag{4}$$

2.3 Item costs for reinforced concrete works

The item costs for reinforced concrete works correspond to the total of individual labour costs and item costs of equipment and materials, as shown in Eq. 5.

$$DC_{RCW} = DC_{LC,RCW} + DC_{EM,RCW} \tag{5}$$

2.4 Costs of production for reinforced concrete works

Pro-rated site overheads can be considered by adding a mark-up to the calculated item costs. This step results in the costs of production CP_{RCW} [€] for reinforced concrete works (Eq. 6).

$$CP_{RCW} = DC_{RCW} * \left(1 + \frac{MU_{OHD}}{100}\right) \tag{6}$$

The percentage of the mark-up MU_{OHD} [%] depends on the determination of the costs of the individual items (either strict separation of item costs from overheads or amalgamation of the two categories). The mark-up for site overheads usually ranges from 15 to 30% in the case of strict separation. To account for inaccuracies, uncertainties etc., the calculation should include a cost buffer, which results in the following equation:

$$CP_{RCW,BU} = CP_{RCW} * \left(1 + \frac{BU_{RCW}}{100}\right) \tag{7}$$

The individual parameters are broken down further for the purpose of detailed costing, and the influential factors are considered more thoroughly. The detailed costing exercise is not covered by this paper.

3. Monte-Carlo Simulation

3.1 Monte-Carlo method

A probability function is allocated to selected parameters for the stochastic computation method (see flow chart in Fig. 1). The values for the range are defined whilst considering relevant management and structural conditions. These values may be based, for example, on internal documentation (final costing of projects) or on information provided in the literature (e.g. reference working times for building construction). Results can be improved significantly by adding probability considerations to the computation process. On the basis of the selected input distributions, the relevant values are shown as probability distributions after a number of iterative steps to be defined beforehand. The application of the Monte-Carlo method makes it possible to calculate, for instance, the probability distribution for the production costs of reinforced concrete works. In a freely selectable number of iterative steps, a software (in this case, @RISK) generates random input values, with each of them occurring in accordance with the pre-defined probability functions, and combines these values according to a pre-determined computation rule (i.e. the equations used when applying the deterministic method).

Input parameters include:

- ranges,
- most likely values (as far as possible) and
- distribution functions

3.2 Computation chart for the Monte-Carlo simulation

Fig. 1 includes a flow chart that illustrates the computation of the probability distribution for the production costs of the reinforced concrete works. In this process, asymmetrical triangles were used as the probability functions for the input parameters. In each iterative step, a single value is taken randomly from each distribution (symbolised by arrows) and combined according to the computation rule shown in the chart (using @RISK).

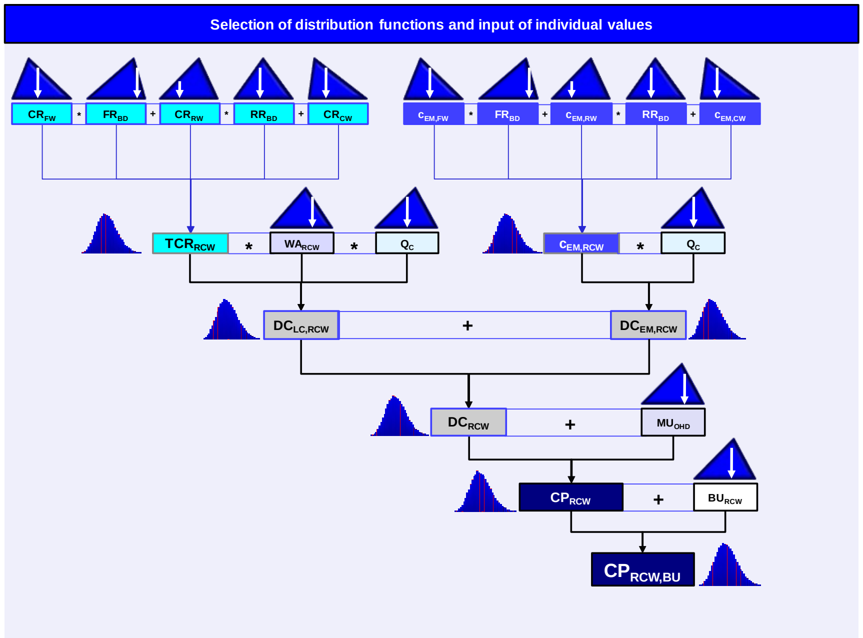

Figure 1: Calculation mode – construction costs: Flow chart (Hofstadler 2010)

At the outset, distribution functions with the corresponding ranges are selected for the individual parameters. The most likely value may also be indicated when using asymmetrical triangles. After defining all input parameters, the number of iterative steps is selected and the simulation launched. The correct number of iterations has been chosen if no marked changes in the output distribution values (e.g. the X_5 and X_{95} quantiles) occur after increasing the number of iterative steps. As an intermediate output, the probability distributions for the total labour consumption rate and the unit costs of labour, equipment and materials are indicated. In addition, the distributions of item costs broken down to labour versus the equipment and materials component are shown. The next distribution function represents the item costs of the reinforced concrete works. The final result shows the costs of production with/without the cost buffer. Intermediate and final results can be analyzed as and when required.

4. Performing arts centre building – Key Details

This paper illustrates the application of the Monte-Carlo simulation using the example of a performing arts centre building.

4.1 Specifications given for the example

Fig. 2 shows an axonometric representation of the building. The building comprises two basement levels, five above-ground storeys and a stage tower. The load-bearing structure is mainly composed of cast-in-situ concrete.

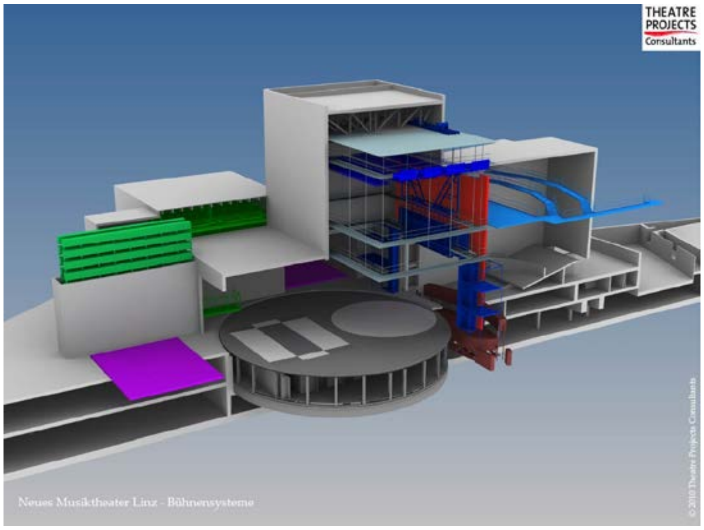

Figure 2: Performing arts centre – Musiktheater Linz [image courtesy of Theatre Projects Consultants].

The maximum length of the building equals approx. 162 metres, its maximum width amounts to about 62 metres. The entire structure has a gross volume of approx. 290,000 m³. The site extends over an area of about 12,000 m², the building has a ground-plan area of approx. 11,000 m². This means that only little space is available outside the building for storage and handling purposes. The formwork area of approx. 127,250 m² and the concrete quantity of 38,275 m³ result in a formwork ratio of approx. 3.3 m²/m³ for the entire structure. The reinforcement ratio is calculated from the reinforcement quantity and the concrete quantity. It amounts to approx. 129 kg/m³.

4.2 Input parameters used

The input values used for the calculations are shown in Table 1. These values were defined on the basis of the existing project documentation whilst considering any identifiable site, management, structural and process conditions. Minimum (MIN), most likely (MLV) and maximum values (MAX) were defined for labour consumption rates, formwork ratio, reinforcement ratio, concrete quantity, equipment and material costs, mark-up etc.

Table 1: Input values to calculate the construction costs

	MIN	MLV	MAX
Average labour consumption rate - formwork placement	1.10 wh/m²	1.20 wh/m²	1.35 wh/m²
Average formwork ratio for the entire building	3.00 m²/m³	3.20 m²/m³	3.50 m²/m³
Average labour consumption rate - reinforcement works	8.50 wh/t	9.50 wh/t	11.00 wh/t
Average reinforcement ratio for the entire building	125.00 kg/m³	129.00 kg/m³	135.00 kg/m³
Average labour consumption rate - concrete works	0.60 wh/m³	0.65 wh/m³	0.75 wh/m³
Concrete quantity	38,000 m³	38,275 m³	39,500 m³
Equipment and materials costs - formwork	7.00 €/m²	8.00 €/m²	10.00 €/m²
Equipment and materials costs - reinforcement	600.00 €/t	630.00 €/t	700.00 €/t
Equipment and materials costs - concrete	80.00 €/m³	90.00 €/m³	95.00 €/m³
Average wage	31.00 €/wh	32.00 €/wh	34.00 €/wh
Mark-up for overheads	10.00 %	14.00 %	17.00 %
Buffer	5.00 %	10.00 %	15.00 %

5. Computation of probability distribution using asymmetrical triangular distributions

Initially, asymmetrical triangles are chosen as the distribution functions for the Monte-Carlo simulation, and the values stated in Table 1 are used for the purpose of calculating the production costs of the reinforced concrete works. 50,000 iterative steps are performed by the @RISK program for a single calculation sequence. At this number of iterations, no significant changes occur in the quantiles (X_5 and X_{95}) when the number of iterative steps is increased further. The computation rule for calculating the production costs is represented by the sequence shown in Fig. 1. In the @RISK program, three values are put in for each parameter. The construction costs of reinforced concrete works are calculated using the mode shown in Fig. 1. The values from Table 2 are used for the input variables to calculate the total labour consumption rate, the unit costs and the costs of production (weighted triangular distribution). 50,000 iterative steps were performed in the @RISK program to calculate construction costs.

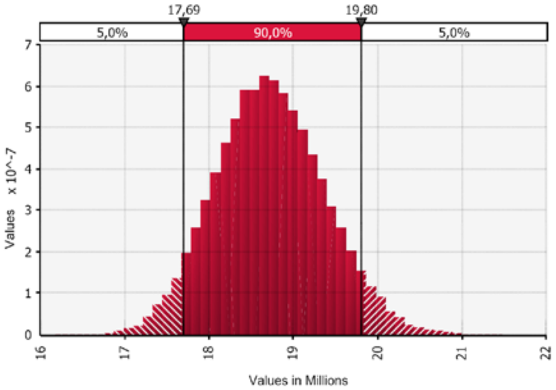

Figure 3: Probability distribution for construction costs, including contingencies

Fig. 3 shows the calculation results for the costs (including buffer) as a probability distribution. This probability distribution can be used to determine the probability of occurrence for selected values. If, for example, it was specified internally that the probability of occurrence must at least be equal to 40% for the costs, the corresponding value is easy to determine. The production costs of reinforced concrete works are below 17,686,532 € only in 5% of all cases (X_5); they exceed 19,797,314 € in 5% of all cases (X_{95}). The range between these quantiles amounts to 2,110,782 €. In 80% of all cases, production costs range from 17,906,000 € to 19,554,840 € - the range is reduced to 1,648,840 €. The expected mean value amounts to 18,720,795 €, with a standard deviation of 638,537 €. The distribution of the values is roughly symmetrical (skewness = 0.111). Median and mode are close to the mean value, which increases its capability of being used as a reliable estimate of the average production costs including buffer.

6. Influence of selected distribution functions on production costs

The following types of distribution were chosen to evaluate the influence of the type of the selected distribution functions on the probability distribution to be calculated:

- isosceles triangle distribution (Calculation 1)
- asymmetrical triangle distribution (Calculation 2)
- rectangular distribution (Calculation 3)
- beta distribution (Calculation 4)

Most likely values are not considered in Calculations 1 (isosceles triangles), 3 (rectangular distributions) and 4 (beta distributions). Asymmetrical triangle distributions are used for Calculation 2. The applied weighting makes it possible to consider the estimated most likely values.

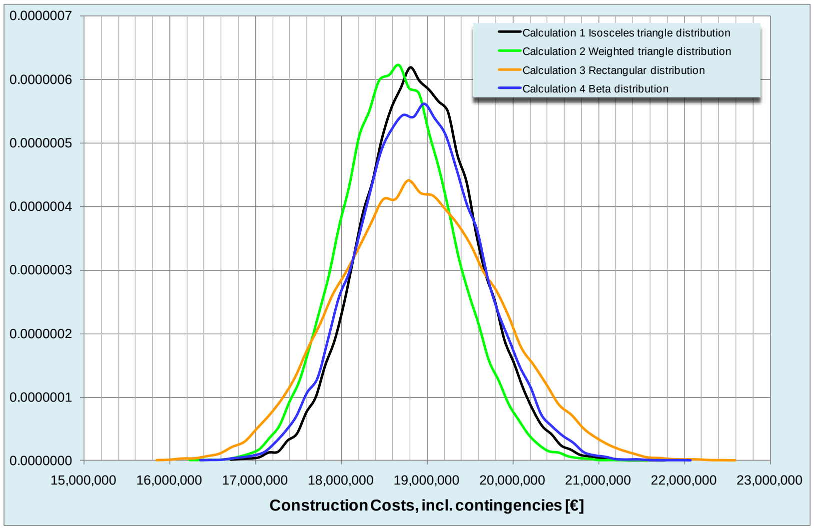

Figure 4: Comparison of simulation results for four different distribution functions – 50,000 iterative steps per simulation

Fig. 4 compares the probability distributions for the production costs including buffer in a diagram. The curves are labelled with the corresponding calculation mode. The following section outlines the differences for selected characteristic values. All four distributions roughly correspond to a normal distribution pattern. Skewness is always positive (rightward skew); it is largest for the asymmetrical triangle distributions (0.111). Kurtosis ranges from 2.827 (rectangular distributions) to 2.921 (asymmetrical triangles).

7. Reduction in the ranges of input distributions

The input parameter ranges are reduced in order to investigate the effects on the distribution functions. Compared to the initial calculation, the ranges of all input distributions are reduced by 15% in a first step and by 30% in a second step (relative to the initial calculation in each of these steps). This method aims to identify the effects on selected parameters. The key question to be answered in this respect is whether the individual types of distribution selected respond to the range reduction to a varying extent. By way of example, the calculation of a 30% range reduction relative to the initial calculation is demonstrated for the concrete quantity. In the initial calculation, the concrete quantity range amounts to 1,500 m³. Following the application of the reduction factor, the modified range amounts to 1,050 m³ (= 1,500 m³ * (1 – 0.3)). In the case of Calculation 2 using asymmetrical triangles, the most likely value remains unchanged for this evaluation. On the other hand, there is always the option of adjusting the most likely value to the range reduction (which was not performed in this exercise). Fig. 5 shows the probability distributions both for the initial calculations and for the calculations performed after the range reductions. The solid lines represent the initial calculations. The key to the diagram shows the allocation of each line to the respective calculation. The dashed lines represent the results after the 15% range reduction; the dotted lines show the outcomes of the 30% reduction. In the case of the standard deviations and 90% spreads, the differences are roughly in line with the pre-defined reductions.

For instance, the use of rectangular distributions (Calculation 3) after a 30% range reduction results in a difference of -30.04% for the standard deviation and in a difference of -30.07% for the 90% spread. Reductions are smallest when applying the beta distributions: the differences amount to -29.68% for the standard deviation and also -29.68% for the 90% spread. Very minor differences were found for the mean and median values in the individual calculations.

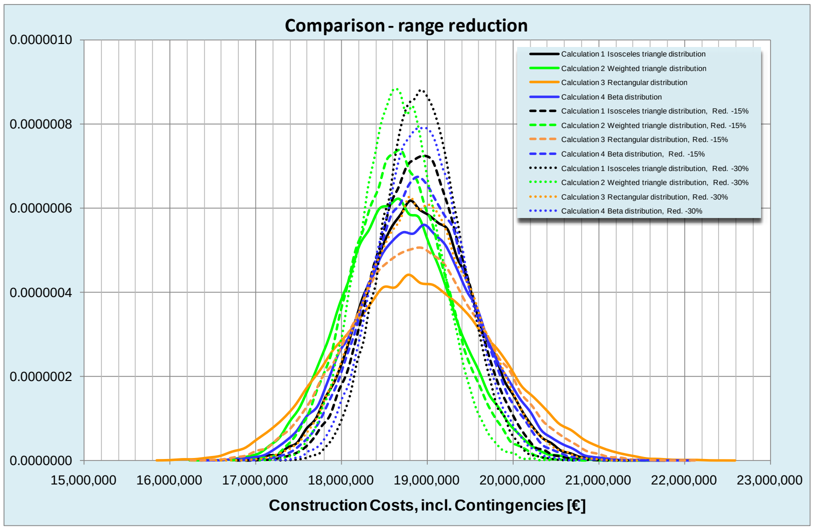

Figure 5: Influence of the reduction in the input value ranges on the probability distribution of production costs (including buffer)

These differences range from -0.00001 to 0.062%. A marked modal value difference of 1.69% occurred between the initial calculation and the 30% reduction in Calculation 3 (beta distribution). Likewise, minor differences were found for the X_5 and X_{95} quantiles. The greatest deviations were documented for Calculation 3 in the case of the 30% reduction (approx. 2.48% for X_5 and -2.26% for X_{95}). The range reduction hardly influences kurtosis whereas skewness is influenced in all calculations. The greatest changes were found for the 15% reduction in Calculation 1 (-40.47%) and for the 15% reduction in Calculation 2 (-34.69%). The smallest change occurred in Calculation 3 (-1.44%) when applying the 15% reduction.

8. Summary

Production costs for reinforced concrete works can be calculated from the preliminary to the detailed costing stage. At the preliminary stage, influential parameters are added up to arrive at only a few key metrics. In the detailed costing exercise, influential parameters are broken down to the process level and individual work sections. Depending on the degree of detail of the costing exercise, the magnitudes of the ranges also change. The higher the accuracy to which the influences of construction processes and structural, site and management conditions are analyzed and evaluated, the narrower the ranges can usually be set. Narrower spreads of input values also have a positive effect on the probability distribution of the relevant output parameters (in this case, production costs). Thus the plausibility and reliability of the results increases in line with the accumulated knowledge and expertise regarding the relevant construction management and economic influences that can be derived from the project to be calculated.

9. References

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