Seventh International Conference on Construction in the 21st Century (CITC-VII)
"Challenges in Innovation, Integration and Collaboration in Construction & Engineering"
December 19-21, 2013, Bangkok, Thailand

Suggested Formula to Predict Concrete Compressive Strength

Muhammad Masood Rafi
NED University of Engineering & Technology
rafi-m@neduet.edu.pk

Muhammad Murtza
NED University of Engineering & Technology

Rizwan U. Farooqui
NED University of Engineering & Technology
rizulhak@neduet.edu.pk

Muhammad Kashif
Lucky Cement Limited
kashif.muhammad@lucky-cement.com

Abstract
Prediction of concrete strength ahead of required age of 28 days can help in reducing waiting time associated with reinforced concrete (RC) construction. This paper presents the details of a strength prediction model formula which was developed to predict 28-day concrete compressive strength. The data of observed concrete strength in the existing literature have been employed in the development and validation of the models. The model takes the effects of cement chemical composition and fineness into account. The predictions are made by using 7-day concrete strength. The proposed model provided good correlation with the observed concrete strength data available in the literature.

Keywords
compressive strength; regression analysis; chemical composition; prediction formula; fineness of cement.

1. Introduction

Cement plays a vital role in the concrete matrix. It acts as a binding agent when mixed with water. Cement consists mainly of silicates and aluminates of lime. The properties of concrete are dependent on the quality of cement, to a large extent. The most valuable property of concrete is its compressive strength which is a function of: (a) properties of aggregates; (b) strength of cement paste and (c) paste aggregate bond strength.

The properties of concrete are generally co-related with its compressive strength; better compressive strength implies enhanced concrete properties. The theory of reinforced concrete (RC) design is based on the 28-day concrete compressive strength which is determined by testing the specimens in the laboratory. ASTM C192/C192M-02 (2002) recommends the use of cylindrical specimens for this purpose.

Trial mixes are usually made in the laboratory and are tested at 28 days. This is a time taking process which requires long waiting time. In order to overcome this hurdle to speedy construction of RC structures, strength prediction formulas have been suggested by various researchers (Gonnerman 1934; Blaine et al. 1968; Popovics 1967a, 1967b, 1981, 1990; Goral 1956; Neville 1966; Zain and Abd 2009; Hasan and Kabir 2011; Abrams 1918; Zielinszki 1909; Kheder et al. 2003; Sang et al. 2003; Akkurt et al. 2004; Hamid-Zadeh et al. 2006; Zain et al. 2010). However, these models have limited application in the construction industry as these are heavily research oriented. Popovics (1998) have discussed in detail limitations of the existing predictive formulas.

This paper presents the details of a prediction formula which has been suggested for the prediction of 28-day concrete compressive strength. The model has been validated using the data of observed concrete compressive strength available in the literature. The simple form of this formula ensures that it can be conveniently used by the professionals and practitioners associated with the construction industry.

2. Review of Existing Prediction Models

The dependence of concrete strength on its water cement ratio was recognized in the twentieth century. Studies by Zielinszki and Zhuk (1901) and Zielinski (1909) investigated the relationship between strength and water-cement ratio. There are several formulae available to predict concrete strength (Popovics 1998). Most of these formulae attempt to estimate strength of comparable concretes from water-cement ratio.

Abrams (1919) suggested a formulation which is widely known as Abrams' formula [Eq. (1)]. The Abrams formula implies that as long as the water-cement ratio of comparable concretes are identical their strengths would be the same. In other words, only the quality of cement paste controls the strength of comparable concretes which remains unaffected by the quantity of cement, cement paste and aggregates.

$$f = A / (B^{w/c}) \tag{1}$$

where f is the concrete strength; w/c = water-cement ratio; and A and B are the empirical parameters which are independent of strength and water cement ratio.

Several augmentations, to include other parameters in Eq. (1), have been suggested. Popovics (1990) suggested the following general augmentation [Eq. (2)].

$$Log\ f=b_0+b_1w/c+b_2c+b_3w+b_4(w+c)+b_5logc+b_6w^2/c+b_7(w/c)^2+b_8C+b_9logC \tag{2}$$

where C is the measure of consistency (slump, vebe time, etc.); and b_i are the values of experimental parameters.

By putting coefficients equal to zero, mathematical models were obtained by Popovics (1990); some of these are given in Eqs. (3) to (5).

$$Log\ f = 4.16 – 0.8586\ w/c \tag{3}$$
$$Log\ f = 4.17 – 1.374\ w/c – 0.00052c \tag{4}$$
$$Log\ f = 4.43 – 0.792\ w/c – 0.00111w \tag{5}$$

Popovics (1990) also suggested a linear model [Eq. (6)], as an augmentation to Eq. (1).

$$f = B_0 + B_1w/c + B_2c + B_3w + B_4(w+c) + B_5\ C +B_6logC \tag{6}$$

Eq. (6) can be transformed into various forms by putting different Bi coefficients equal to zero.

In addition, Popovics (1990) suggested Eq. (7) for the determination of threshold cement content (c_{th}), beyond which the concrete strength (f_{max}) does not increase by cement addition despite a reduction in w/c.

$$c_{th} = 50 \sqrt{w} \tag{7}$$

$$\log f_{max} = 4.71 - 0.0535 \sqrt{w} \tag{8}$$

Note water content (w) in Eqs. (7) and (8) are taken in lb/yd^3

Popovics (1998) suggested a model which is applicable to all Portland cement types, a wide range of curing temperatures, and age between 1 day and 1 year. The strength development is considered a two-stage process. The first stage corresponds to a period when hydration is controlled by the rate of chemical reactions [Eq. (9)] whereas hydration in the second stage is controlled by a diffusion mechanism [Eq. (10)].

$$f_{ct} = (f_o f_{28}) \times (100 - C_3) \times e^{-a1(t - ts) \times (S / 300)} - (100 - C_3) \times e^{-a2(t - ts) \times (S / 300)}) / (100 - C_3 \times e^{-a1(28 - ts) \times (S/300)} - (100 - C_3) \times e^{-a2(28 - ts) \times (S/300)}) \tag{9}$$

$$f_{dt} = \omega \times \log(t/t_d) \tag{10}$$

where t is the age of concrete (or mortar) at the time of testing (days) (ts < t < td); f_{ct} is the strength of concrete developing during the first stage; f28 is the strength of concrete of the same composition at 28 days but made with Type-I Portland cement and cured for 28 days in the standard manner at 22.8oC; f_o is the hypothetical strength that the concrete would achieve if the control of the strength development by the rate of chemical reactions did not stop at td but rather continued indefinitely expressed as percent/100 of the 28-day strength; C_3 is the computed C_3S content of the Portland cement (percent); a1 and a2 are the rate parameters (1/day); S is the Blaine specific surface of the cement (m^2/kg) (specific surface of a Type I Portland cement is assumed to be 300 m^2/kg); t_d is the age when the first stage of strength development is assumed to end, and the second stage starts (days); and ω is the experimental parameter that may be a function of the cement composition but appears to be independent of the curing temperature.

Popovics and Ujhelyi (2008) proposed a modified form of Eq. (1) [Eq. (11)] which considers the effect of varying cement content on strength with or without air content along with water-cement ratio.

$$f = (A / B^{(w/c + Ec)}) \times 10^{-\gamma a} \tag{11}$$

where E is the empirical parameter; c is the cement content; a is the air content (macro porosity) % by volume; and γ is the experimental parameter that is independent of the strength and age of concrete within practical limits but depends on the type of strength

Zain and Abd (2009) suggested Eq. (12) for strength prediction of concrete. The authors concluded that the concept of using early age strength to predict strength at later ages can be relied upon. The values of a_i parameters are different for test at different ages.

$$f_k = a_0 C^{a1} W^{a2} FA^{a3} CA^{a4} \rho^{a5} (w/c)^{a6} \tag{12}$$

where k is taken as 7 or 28 days; a_i is the regression coefficients which are different for 7 and 28 days; C is the cement content (kg/m^3); W is the water content (kg/m^3); FA is the fine aggregates (kg/m^3); CA is the coarse aggregate (kg/m^3); ρ is the density (kg/m3); and f_k is the compressive strength

Hasan and Kabir (2011) suggested Eq. (13) to predict strength of concrete.

$$St_n = p.D_n / (D_n + q) \quad (13)$$

where S_{tn} is the strength of concrete at nth day (n=1, 2, 3,….); D_n is the number of days; a, b, c, d and e are the coefficients; and p and q are constants [Eq. (14)]

$$p = a + b.q + c.S_{tn} + d.q.S_{tn} + e.(S_{tn})^2 \quad (14)$$

Onwuka et al. (2011) suggested a mathematical method [Eq. (15)] based on modified regression theory for the prediction of concrete strength. The model can prescribe all the mixes that will produce a desired strength of concrete. It can also predict the strength of concrete if the mix proportions are specified.

$$Y = \Sigma\alpha_i z_i + \Sigma\alpha_{ij} z_i z_j \quad (15)$$

where Y is the response function; z_i is the predictor; and α_i is the coefficient of the optimization model equations

It can be seen from the above that all the available formulae employ several parameters and coefficients which need to be determined separately. This factor makes the use of these formulae difficult for the construction industry.

3. Suggested Predictive Formula

The suggested formula for the prediction of concrete 28-day strength is based on 7-day strength of concrete so as to avoid the use of multiple variables. Note the concrete strength is influenced by compound composition (in particular C3S) and fineness of cement largely up to 7 days and their influence becomes small at later ages. As a result, the formula takes the effects of chemical and compound composition, and fineness of cement, and mix proportion of concrete. The prediction formula is of the form as given in Eq. (16).

$$f_c = \alpha \times f_{c,7} \times t_n^{\beta}; \{7 < t_n \leq 28\} \quad (16)$$

where f_c,7 is the 7-day compressive strength of concrete; t_n is the age of concrete at which strength is to be predicted (n = 8,….,28); and α and β are the dimensionless factors

The value of α and β is dependent on the cement properties such as chemical and compound composition, and fineness. Rafi and Nasir (2013) carried out studies to compare the strengths of comparable concrete mixes. Seven brands of cements available in Pakistan were employed in these studies. Concrete cylinders of three strength classes were tested. These include 21 MPa (M21), 34 MPa (M34) and 48 MPa (M48). A summary of the results of observed strength is given in Table 1 which compares the concrete strength at 7, 14 and 28 days of the tested cylinders. A regression analysis has been carried out by using the method of nonlinear least squares to determine the values of α and β for the cement brands employed by Rafi and Nasir (2013) (Table 1). The details of this work will be presented separately. The final equation obtained is given as eq. (17). Note that some of the aforementioned methods of strength prediction were also tried on the data in table 1 and it was found that these do not predict the concrete strength adequately.

$$f_c = 0.56 \times f_{c,7} \times t_n^{0.29}; \{7 < t_n \leq 28\} \quad (17)$$

Table 1: Observed compressive strengths of mixes (Rafi and Nasir 2013)

Cement brand	Mix type	Compressive strength (MPa)		
		7-day	14-day	28-day

	M21	15.02	18.00	20.17
A	M34	29.16	32.74	35.68
	M48	33.59	41.34	48.10
	M21	13.72	18.53	21.32
B	M34	28.14	34.18	36.28
	M48	31.96	44.67	48.82
	M21	11.81	16.23	18.95
C	M34	23.12	28.06	32.50
	M48	30.16	36.61	45.89
	M21	11.86	14.58	18.91
D	M34	21.14	24.27	31.49
	M48	29.37	34.46	44.86
	M21	11.74	14.15	18.50
E	M34	20.64	23.82	31.10
	M48	30.19	34.33	43.74
	M21	11.57	14.47	18.60
F	M34	20.70	23.94	31.48
	M48	29.64	34.46	44.57
	M21	11.64	13.88	17.85
G	M34	197.76	23.17	30.47
	M48	28.10	33.31	43.43

Figure 1 illustrates the results of observed and predicted strengths using Eq. (17) at 14 and 28 days. It is seen in Figure 1 that the two data match closely for most of the concrete strength at both ages; the maximum error at any age is less than 10%. As a result, Eq. (17) can be employed to predict the strength of concrete made with the cement brands manufactured in Pakistan.

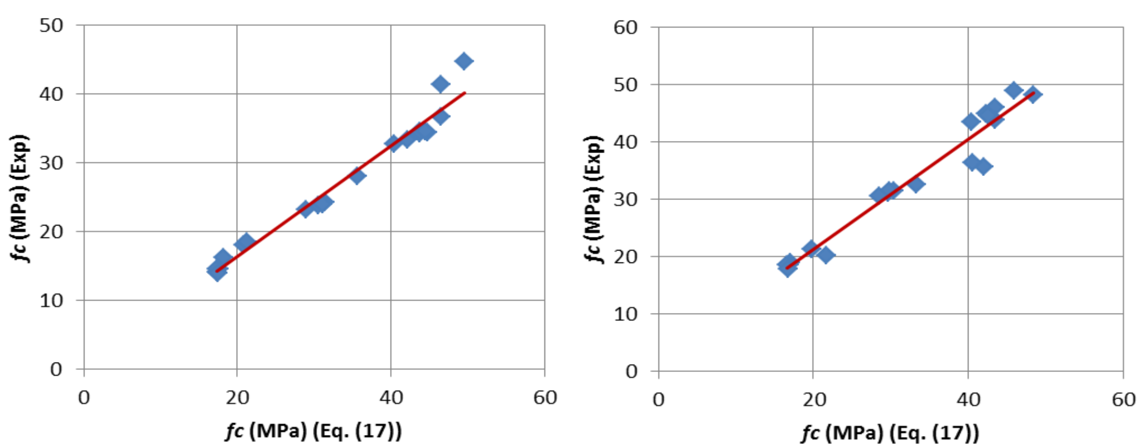

Figure 1: Observed and predicted concrete compressive strength: (a) 14-days; (b) 28-days

The suggested equation (Eq. (16)) was further validated with the help of data in the available literature. Table 2 shows the details of mixes along with their 7-day and 28-day (fc,28) strengths and cement brands employed by Naseer and Al-Manaseer (1987). The values of the parameters α and β for these data come out to be 0.59 and 0.23, respectively. Table 2 also provides the ratio of reported and predicted 28-day strengths. It is noted in Table 2 that the difference between the reported and predicted strengths is within 10% and is insignificant.

Table 2: Details of mixes and compressive strength (Naseer and Al-Manaseer 1987)

Cement brand	Mixes	Cement (Kg/m^3)	FA (Kg/m^3)	CA (Kg/m^3)	W/C	$f_{c,7}$ (MPa)	$f_{c,28}$ (MPa) Reported (A)	Predicted (B)	A/B
Type-I Portland cement	Mix 1	310	800	1100	0.55	19.59	27.59	24.99	1.09
	Mix 1	310	800	1100	0.55	22.00	25.86	28.08	0.98
	Mix 1	310	800	1100	0.55	22.28	26.21	28.43	0.91
	Mix 1	310	800	1100	0.55	20.83	26.34	26.58	0.99
	Mix 2	310	800	1100	0.55	19.31	25.45	24.64	1.07
	Mix 2	310	800	1100	0.55	18.83	23.52	24.03	1.06
	Mix 3	310	800	1100	0.55	19.24	27.95	24.55	0.96
	-	-	-	-	-	20.76	27.59	26.49	1.06
	-	-	-	-	-	22.00	26.25	28.08	0.98
	-	-	-	-	-	20.85	25.87	26.61	0.99
	-	-	-	-	-	22.25	27.59	28.39	0.91

4. Conclusions

This paper presented a formula to predict concrete compressive strength. The formula uses 7-day strength to predict concrete strength up to 28 days and takes the effects of chemical and compound composition, and fineness of cement. The suggested formula made reliable prediction for strength of concrete made with the cements available in Pakistan. The formula was also validated using the data of concrete compressive strength available in the literature and satisfactory results were noted.

5. Acknowledgments

The authors wish to acknowledge the financial assistance provided by Lucky Cement.

6. References

Abrams, D. A., “Design of Concrete Mixtures,” Bulletin 1, Structural Materials Research Laboratory, Lewis Institute, Chicago, 1918.

Abrams D. A., “Deign of Concrete Mixtures”, Bulletin 1, Structural Materials Research Laboratory, Lewis Institute, Chicago, 1919.

Akkurt S., Tayfur G., Can S., “Fuzzy Logic Model for Prediction of Cement Compressive Strength”, Cement Concrete Research, Vol. 34, 2004, pp. 1429-1433.

American Society for Testing Materials (ASTM), “Standard Practice for Making and Curing Concrete Test Specimens in the Laboratory (ASTM C192/C192M-02),” ASTM International, West Conshohocken, PA, 2002.

Blaine, R. L., Arni, H. T., and Defore, M. R., “Compressive strength of test mortars. Interrelations Between Cement and Concrete Properties,” Part 3, Building Science Series 8, National Bureau of Standards, Washington, 1968, pp. 1-65.

Gonnerman, H. F., “Study of cement composition in relation to strength, length changes, resistance of sulfate waters and to freezing and thawing of mortars and concrete.” Proceedings ASTM, V. 34, No. II, 1934, pp. 244-295.

Goral, M. L., “Empirical time-strength relations of concrete.” ACI Journal Proceedings, V. 53, No. 2, 1956, pp. 215-224.

Hamid-Zadeh N., Jamil A., Nariman-Zadeh N., and Akberzadeh H., “A Polynomial Model for Concrete Compressive Strength Prediction Using GMDH-type Neural Networks and Genetic Algorithm.” Proc., 5th WSEAS International Conference on Systems Science and Simulation in Engineering, Canary-Island, Spain, 2006, pp. 13-18.

Hasan M. M., and Kabir A., “Prediction of Compressive Strength of Concrete from Early Age Test Result.” Proc., of 4th Annual Paper Meeting and 1st Civil Engineering Congress, Dhaka, Bangladesh, 2011.

Kheder G. F., Al-Gabban A.M., and Suhad M.A., “mathematical model for the prediction of cement compressive strength at the ages of 7 & 28 days within 24 hour.” Materials and Structures, V. 36, No. 264, 2003, pp.693-701.

Naseer K. W., Al-Manaseer A. A., “ It’s time for a change from 6x12 to 3x9-in. cylinders.” ACI Materials Journal, V. 84, No. 3, 1987, pp. 213-216.

Neville, A M., “A general relation for strengths of concrete specimens of different shapes and sizes.” ACI Journal Proceedings, V. 63, No. 10, 1966, pp. 1095-1109.

Onwuka D. O., Okere C. E., Arimanwa J. I., and Onwuka S. U., “Prediction of concrete mix ratio using modified regression theory.” Computational Methods in Civil Engineering, V. 2, No. 1, 2011, pp. 95-107.

Popovics, S., “Generalization of the Abrams kaw-predication of strength development of concrete from cement properties.” ACI Journal Proceedings, V. 78, No. 2, 1981, pp. 123-129.

Popovics, S., “Analysis of the concrete strength versus water-cement ratio relationship.” ACI Materials Journal, V. 87, No. 5, 1990, pp. 517-529.

Popovics, S., “Discussion of “General relation for strengths of concrete specimens of different shapes and sizes” by A. M. Neville.” ACI Journal Proceedings, V. 64, No. 6, 1967a, pp. 1566-1568.

Popovics, S., “Factors affecting the relationship between strength and water-cement ratio.” Materials Research and Standards, MTRSA, V. 7, No. 12, 1967b, pp. 527-534.

Popovics S., “Strength and Related Properties of Concrete”, Wiely, New York, 1998.

Popovics S., Ujhelyi J., “Contribution to the Concrete Strength versus Water-Cement Ratio Relationship”, Journal of Materials in Civil Engineering, Vol. 20, No. 7, 2008, pp. 459-463.

Rafi MM, Nasir MM. (2013). Experimental Investigation of Chemical and Physical Properties of Cements Manufactured In Pakistan. ASTM Journal of Testing and Evaluation 42(3), 2013, DOI: 10.1520/JTE20130158.

Sang H. H., Jin K. K., and Yon D. P., “Prediction of compressive strength of fly ash concrete by new apparent activation energy function.” Cement Concrete Research, V. 33, No. 7, 2003, pp. 965-971.

Zain M. F. M., and Abd S. M., “Multiple regression model for compressive strength prediction of high-performance concrete.” Journal of Applied Sciences, V. 9, 2009, pp. 155-160.

Zain, M. F. M., Suhad M., Abd-Hamid R., and Jamil M., “Potential for utilizing concrete mix properties to predict strength at different ages.” Journal of Applied Sciences, V. 10, No. 22, 2010, pp. 2831-2838.

Zielinszki, S., “The Development of the Setting of Roman and Portland Cements in Pastes, in Mortars, and in Concrete.” Proc., International Association for Testing Materials, Copenhagen, 1909, pp. 1-55.

Zielinszki S., Zhuk J., “ Comparative Investigation of Roman Cements.”, Proc., Third Int. Congress of the Assoc. for Testing Materials, 1901.