Fourth International Conference on Construction in the 21st Century (CITC-IV)
"Accelerating Innovation in Engineering, Management and Technology"
July 11-13, 2007, Gold Coast, Australia

Long Term Multiobjective Project Selection Under Uncertainty

José A. Faria, Ph.D., PMP, A.M. ASCE
Assistant Professor, Florida International University, Miami, Florida, USA

Thomas R. Beranek, PMP
Director of Medical Facilities Programs, USAF, Alexandria, VA, USA

Beste Gul Durmus
Graduate Student, Florida International University, Miami, Florida, USA

Abstract
Project selection has traditionally been a difficult decision making problem especially for large organizations with many stakeholders. The basic problem is to reach an agreement among the different stakeholders to allocate limited resources to fund multi-year capital projects in some optimal fashion. Considering that there is uncertainty about the final cost of each project, the organization must try to prevent exceeding the budget when making the decision. The authors present a framework using the Analytic Hierarchy Process (AHP) to reach an agreement between stakeholders about the relative importance of funding the different projects available during the upcoming years. A multi-year knapsack optimization problem is solved using an equivalent non-linear deterministic constraint in lieu of the probabilistic constraint required to maintain a given confidence level of not exceeding each year's available budget. The framework is explained and presented with an example drawn from an agency whose main forte is managing more than 80 facilities worldwide and allocating billions of dollars to operate, maintain, replace or upgrade these facilities over the years.

Keywords
AHP, Multiobjective, Project Selection, Stochastic, Knapsack.

1. INTRODUCTION

Some large organizations have a competing set of projects to undertake, but because the yearly available budget is limited, only a subset of the available projects can be undertaken at a given time. The projects may vary in size from a few thousand dollars to hundreds of millions of dollars, and in duration from one to several years. Because the cost of the projects will increase due to inflation, some projects might become too expensive to be selected in later years. Therefore, the decision makers, who allocate the funds, have the task of analyzing the different aspects of each project before making a decision. They should identify their priorities, and give each project a weight according to each project's aspects and the potential time of execution.

We present a multiobjective optimization model to account for the benefits, uncertainties and budget limitations involved in project selection. The paper first presents a literature review and the underlying theory required for the model, followed by a case study that illustrates the procedure, finishing with the conclusions.

2. LITERATURE REVIEW

The project selection problem under uncertainty has been widely studied since the 1950's when the publication of the paper "Portfolio Selection" by the Nobel Price winner Harry Markowitz provided a stepping stone into the subject (Markowitz, 1952). Markowitz proposed an optimization method whose objective was to minimize the variance of the portfolio (and therefore the risk) subject to a budget constraint and expected minimum returns. By changing the returns expectations one can find a series of points that form the so-called "efficient frontier" or Pareto optimal curve. This curve is such that each point is optimal in the sense that is not possible to obtain a solution that is better in any of the objectives without deteriorating the value of at least another objective (Steuer 1986).

Although Markowitz's work was aimed at stock market decisions, other authors have quickly expanded to a myriad of applications. Some authors have focused on finding the combination of projects that provides the lowest total cost for a given probability of success (Kujawski, 2002) while others have applied simulation methods to accommodate the dynamics of stock trading (Detemple, et al., 2003).

A number of publications have been made in recent years focusing on evaluating engineering, procurement and construction projects based on an established set of objectives. These publications have generally considered either multiple objectives without risk considerations, or a single objective with some form of risk assessment. Moselhi and Deb (1993) presented a comprehensive methodology for the consideration of multi-objectives and risk in the selection construction projects.

Some deterministic approaches have been taken based on the well-known "knapsack problem" (Winston 2004, and Badiru and Pulat 1995), the budget available to pursue the project portfolio is viewed as a knapsack which has a capacity equal to the budget amount, each project is view as taking up space in proportion to it's cost. These approaches include among others the work by Mandakovic et al. (1985) and Gori (1996).

The project selection for Resource and Development (R&D) projects has had a fair share of attention. Williams (1969) worked evaluation and selection of R&D projects for the British Aircraft Corporation where the benefit of the projects was combined as a weighted sum of factor scores. Lockett and Freeman (1970) created a framework to account for the stochastic nature of resource requirements and project benefits, using a combination of probabilistic networks, simulation and mathematical programming. Taylor et al. (1982) extended the traditional integer programming approach to consider other non linearities that result from resource allocation. Stewart (1991) developed a decision support system for the selection of a portfolio of R&D projects which was carried out for a large electricity utility corporation where he treated the problem as a multi-criteria decision problem. The application of this approach did require a less usual form of scalarizing function as well as a heuristic algorithm for solving a non-linear knapsack problem.

Some applications of multiobjective optimization applied to medical facilities, include the work of Argote (1982) who studied the relationships among uncertainty, coordination, organizational effectiveness of hospital emergency units. Although this work was not a project selection work per se, it was applied to medical facilities.

The use of the analytic hierarchy process to measure the initial viability of industrial projects was presented by Alidi (1996) where he emphasized the importance in ranking the projects for the efficient allocation of the company's resources. The AHP was also used in risk assessment in construction by Mustafa and Al-Bahar (1991) in order to analyze and assess project risks during the bidding stage of a construction project and to overcome the limitations of the approaches currently used by contractors. Cheng and Li (2001) used AHP to prioritize different forms of information for better resource allocation in construction projects. On the other hand, in 2005, they implemented another method, Analytical

Network Process (ANP), to deal with interdependent relationships within a multicriteria decision-making model where they prioritized a set of projects by using a five-level project selection model.

A Multiobjective optimization approach with stochastic project cost was analyzed by Gabriel et al. (2005, 2006) using chance constraints to maintain the risk of exceeding the budget within limits. Li and Puyan (2006) deals with the stochastic optimization of highway projects under budget uncertainty, they formulated a stochastic Knapsack problem with Ω-stage budget recourses.

Our work builds on the deterministic equivalent formulation presented in Gabriel et al. (2006) but expands by introducing multiple year consideration with the effect of inflation, and presents the case of an agency whose main forte is managing medical facility programming, planning, design, construction, maintenance, and sustainment.

3. DECISION MAKING CONSIDERATIONS

When a large number of projects are available and one is faced with the task of selecting among them the “best” to be funded, many aspects come into consideration. The first one is perhaps the cost, so it is necessary that the total cost of the selected projects not exceeds the available budget. From this perspective one way of obtaining the best solution is to use the knapsack problem. The problem can be stated as: which projects should be included to maximize some objective function of importance to the decision maker, subject to the limitations imposed by yearly budget? When dealing with a multi-year problem or long term planning, there is a budget allocated for each year and the selection should not exceed the budget given for each year.

3.1 Benefits of the Portfolio

One function to measure benefit can be computed as the sum of priorities given to the projects selected in the portfolio. The decision maker could assign one priority to each project independently of the year, or a different priority depending on the year where the project is selected for funding. The values for the priorities used can be found by using the Analytic Hierarchy Process (AHP) (Saaty 1980). This process is based on a pairwise comparison of the attributes to determine the relative importance between the projects according to the judgment of the decision makers. If the projects are given the same priority regardless of the year in which they are chosen, then two solutions that select the same projects, would have the same benefit regardless of when the projects are selected. This might not have much practical sense, for example selecting all projects in the first year would be considered equally beneficial as selecting all projects in the forth year and do nothing the first three years.

For example consider the case of five projects to be funded in a five year window. To give each project a different priority, the decision makers can agree in a rank similar as presented in Table 1. The decision maker has given to each project a different rank or benefit depending on the year that each project is funded. The ranks are decreasing each year to favor earlier selection of the projects.

Table 1 Matrix of ranks for each project per year

	Year 1	Year 2	Year 3	Year 4	Year 5
p1	0.8110	0.7299	0.6569	0.5912	0.5321
p2	0.6183	0.5565	0.5009	0.4508	0.4057
p3	1.5836	1.4252	1.2827	1.1544	1.0390
p4	0.8303	0.7473	0.6725	0.6053	0.5448
p5	0.9101	0.8191	0.7372	0.6635	0.5971

Some agencies may loose available funds if they are not allocated, so another valid measurement of the benefit would be to minimize the slack or difference between the budget and the funds allocated. This benefit measurement would give priority to those selections that have minimum total slack. Yet another benefit measure could be just the contrary, choose a portfolio that provides maximum slack, the reasoning behind such measure is that because the project costs are stochastic, their final cost might be higher than predicted. Therefore, having extra slack in the selection can accommodate for any unforeseen events that might rise. This measurement is equivalent to the minimum cost, since a portfolio with minimum cost would have the maximum slack. Yet another possible measurement is the well-known net present value (NPV) which would give preference to a portfolio that has a higher net present value as compared to another one. The problem with this measurement is the dependency of the solution on the interest rate chosen. A solution that has a lower NPV for a certain interest rate might have a higher one with a different interest rate so the selection of the interest rate must be carefully considered.

3.2 Expected Cost of the Portfolio

Considering the cost of each project as a normally distributed variable, then the sum of the costs for all projects on each year is also a normally distributed variable. This holds true since the cost of each project is a sum of the cost of many deliverables, each one of them being a random variable, then by the Central Limit Theorem the total cost for each project will tend to be a normally distributed variable (Garvey 2000), and the sum of normally distributed variables is another normally distributed variable.

Because any given project might have a duration that exceeds a year, then the cost for each year and the probability to exceed such cost needs to be computed.
By the properties of the mean and variance of a random variable, if X is a random variable with mean μ and variance v then for any constant c the mean of cX is $c\mu$ and the variance of cX is $c^2 v$ (Winston 2004). We can calculate the expected value and variance of the cost for each project, for every year using this knowledge. If the duration of a project is d_i, and the expected total cost of any given project i is $\bar{c}_i$, then for each year the expected cost of project i is given by $\frac{\bar{c}_i}{d_i}$ and the variance of the cost for the given year is then $\frac{v_i}{d_i^2}$. For those projects whose duration is fractional then the last year will be subject to a smaller cost and variance as compared to the earlier years. Consider for example a project with a duration of 1.5 years, during the first year the cost would be 2/3 of the total cost and the variance would be $(2/3)^2$ of the original variance whereas the last year would have (1/3) of the total cost and $(1/3)^2$ of the total variance. This assumes that the project's expenses are uniform over the duration of the project.

With this in mind one can create a matrix f filled with factors that can be used to distribute the project's cost over the years. In this matrix f an entry $f_{i,j}$ represents the fraction of the cost distributed for project i in year j. The distributed costs for each project over the years can then be computed using a distributed cost matrix formed as follows:

$$(1)\ dc: \text{distributed cost matrix} = \begin{bmatrix} \bar{c}_1 f^{(1)} \\ \bar{c}_2 f^{(2)} \\ \ldots \\ \bar{c}_n f^{(n)} \end{bmatrix}$$

where

$\overline{c_i}$ is the expected cost of project i, and

$f^{(i)}$ is the i^{th} row of the matrix of cost distribution factors

Consider for example the case of five projects with costs and durations as per Table 2

Table 2 List of five projects with their durations

Project	Projects	Cost	Duration
p1	OB Modernization/Surgical Renovation	$15,657,597	1.5 years
p2	OB Modernization	$15,657,597	1 year
p3	Hospital Replacement	$126,065,619	3 years
p4	AHCC Addition/Alteration	$38,820,383	3.5 years
p5	AHCC Replacement	$64,710,595	2.5 years

A corresponding matrix of distribution factors is shown in Table 3

Table 3 Matrix of distribution factors for projects in Table 2

Project	Year 1	Year 2	Year 3	Year 4
p1	2/3	1/3	0	0
p2	1	0	0	0
p3	1/3	1/3	1/3	0
p4	2/7	2/7	2/7	1/7
p5	2/5	2/5	1/5	0

And the distributed cost matrix would be as presented in Table 4.

Table 4 Distributed cost matrix

Project	Year 1	Year 2	Year 3	Year 4
p1	$10,438,398	$5,219,199	$0	$0
p2	$15,657,597	$0	$0	$0
p3	$42,021,873	$42,021,873	$42,021,873	$0
p4	$11,091,538	$11,091,538	$11,091,538	$5,545,769
p5	$25,884,238	$25,884,238	$12,942,119	$0

If a project is selected to start on a given year, then it is assumed that the project will continue over the years until the project is completed and the different years will not be affected by inflation since this factor is included in the initial cost. If the first assumption is not true, then the project can be broken down in phases where each phase can be treated as a separate project. If the second assumption is not true, then the inflation factor can be applied to all the phases of each project rather than to those projects not selected for funding on a given year.

The expected cost of the portfolio during year 1 is equal to the sum of the first year for the projects selected to be funded during the first year. For year 2, the total expected cost of the portfolio is the cost of those projects selected during year one that are still ongoing in the second year, plus the expected cost of the projects selected for funding on the second year after adjusting for inflation.

These costs can be expressed as:

$$(2)\ EC_j = \sum_{k=1}^{j} \left(1+\inf\right)^{(k-1)} \sum_{i=1}^{n} dc_{i,j-k+1} x_{i,k}\,; \forall j = \{1,2,..y\}$$

where
EC_j is the expected cost of the portfolio for year j
n is the number of projects to consider,
y is the number of years in consideration,
$\inf$ is the inflation rate per year,
$dc_{i,j}$ is the distributed cost for project i during year j,
$x_{i,j}$ is the decision variable associated to fund project i during year j

Equation (2) can be expressed in matrix vector form by constructing an expanded distribution cost matrix edc as follows. Starting with the dc matrix we compute $\left(1+\inf\right)^{(j-1)} dc$ for every year and place it below the dc matrix offset by one column obtaining an extended distributed cost matrix as follows:

$$(3)\ edc = \begin{bmatrix} dc_{1,1} & ... & dc_{1,y} & 0 & 0 & 0 \\ ... & ... & ... & 0 & 0 & 0 \\ dc_{n,1} & ... & dc_{n,y} & 0 & 0 & 0 \\ 0 & (1+\inf)dc_{1,1} & ... & (1+\inf)dc_{1,y} & 0 & 0 \\ 0 & ... & ... & ... & 0 & 0 \\ 0 & (1+\inf)dc_{n,1} & ... & (1+\inf)dc_{n,y} & 0 & 0 \\ 0 & 0 & ... & ... & ... & 0 \\ 0 & 0 & 0 & (1+\inf)^{(y-1)} dc_{1,1} & ... & (1+\inf)^{(y-1)} dc_{1,y} \\ 0 & 0 & 0 & ... & ... & ... \\ 0 & 0 & 0 & (1+\inf)^{(y-1)} dc_{n,1} & ... & (1+\inf)^{(y-1)} dc_{n,y} \end{bmatrix}$$

This extended distributed cost matrix allows rewriting (2) in a more succinct vector matrix form as:

$$(4)\ EC = \left[edc^T\right]\left[x\right]$$

where
EC is a column vector with the cost of the portfolio for each year
$\left[x\right]$ is a column vector containing the selection of each project for each year, $\left[x\right]$ is formed taking the decision variables $x_{i,j}$ and placing each column j below the previous one forming one column vector as follows:

$$(5)\ [x] = \begin{bmatrix} x_{1,1} \\ .. \\ x_{5,1} \\ x_{1,2} \\ .. \\ x_{5,2} \\ .. \\ x_{5,1} \\ .. \\ x_{5,5} \end{bmatrix}$$

In the case of our 5 project example, the matrix EC can be written as presented in Table 5.

Table 5 Extended distribution cost matrix for the projects presented in Table 2.

	Year 1	Year 2	Year 3	Year 4	Year 5
1	$10,438,398	$5,219,199	$0	$0	$0
2	$15,657,598	$0	$0	$0	$0
3	$42,021,873	$42,021,873	$42,021,873	$0	$0
4	$11,091,539	$11,091,539	$11,091,539	$5,545,769	$0
5	$25,884,239	$25,884,239	$12,942,120	$0	$0
6	$0	$10,960,318	$5,480,159	$0	$0
7	$0	$16,440,477	$0	$0	$0
8	$0	$44,122,967	$44,122,967	$44,122,967	$0
9	$0	$11,646,116	$11,646,116	$11,646,116	$5,823,058
10	$0	$27,178,451	$27,178,451	$13,589,226	$0
11	$0	$0	$11,508,334	$5,754,167	$0
12	$0	$0	$17,262,501	$0	$0
13	$0	$0	$46,329,115	$46,329,115	$46,329,115
14	$0	$0	$12,228,422	$12,228,422	$12,228,422
15	$0	$0	$28,537,374	$28,537,374	$14,268,687
16	$0	$0	$0	$12,083,751	$6,041,875
17	$0	$0	$0	$18,125,626	$0
18	$0	$0	$0	$48,645,571	$48,645,571
19	$0	$0	$0	$12,839,843	$12,839,843
20	$0	$0	$0	$29,964,242	$29,964,242
21	$0	$0	$0	$0	$12,687,938
22	$0	$0	$0	$0	$19,031,908
23	$0	$0	$0	$0	$51,077,849
24	$0	$0	$0	$0	$13,481,835
25	$0	$0	$0	$0	$31,462,455

Although the extended cost matrix can be calculated until the 8^{th} year, because the projects can extend up to 3.5 years and if selected on the 5^{th} year they will span up to the 8^{th} year, we limited our calculations to the first five years. The reason is that the timeframe for the analysis within the agency is limited for five years, there is no budget set up thereafter. New projects come along over time, and the organization will include them in their updated plan which would be calculated yearly for the next five years.

3.3 Variance of the Portfolio

Similarly to the computation of the distributed cost, the distributed variance for each project over the years can be computed using a distributed variance matrix formed as follows:

$$(6)\ dv : \text{distributed variance matrix} = [v]\left[f^2\right]$$

where:

v is the variance-covariance matrix, and

$\left[f^2\right]$ is the matrix of distribution factors squared

Consider for example the case of the same five projects presented in Table 2, the variance-covariance matrix is presented in Table 5.

Table 6 Variance covariance matrix for five projects

Project	p1	p2	p3	p4	p5
p1	66,970,274,124	0	0	0	0
p2	0	58,087,907,171	0	0	0
p3	0	0	4,341,354,233,402	0	0
p4	0	0	0	411,672,193,471	0
p5	0	0	0	0	1,143,885,864,676

The matrix of distribution factors squared is presented in Table 6

Table 7 Matrix of distribution factors squared

Project	Year 1	Year 2	Year 3	Year 4
p1	4/9	1/9	0	0
p2	1	0	0	0
p3	1/9	1/9	1/9	0
p4	4/49	4/49	4/49	1/49
p5	4/25	4/25	1/25	0

And the distributed variance matrix would be as presented in Table 7.

Table 8 Distributed variance-covariance matrix

Project	Year 1	Year 2	Year 3	Year 4
p1	29,764,566,277	7,441,141,569	0	0
p2	58,087,907,171	0	0	0
p3	482,372,692,600	482,372,692,600	482,372,692,600	0
p4	33,605,893,345	33,605,893,345	33,605,893,345	8,401,473,336
p5	183,021,738,348	183,021,738,348	45,755,434,587	0

For each year, the variance of the portfolio can be calculated given the variance-covariance matrix and the selection of projects for that particular year. For the first year, the variance of the portfolio is given by the sum of the covariances for the projects selected on year one. When the projects are independent, the variance-covariance matrix has entries only in the diagonal such as in Table 5, the calculation of the total variance is be the sum of the individual project cost variances for those projects chosen for funding during the year. In general, the projects do not need to be independent. For the second year the variance of the portfolio is given by the sum of the covariances for the projects selected during the second year, plus the sum of the covariances for the ongoing projects selected on the previous year.

For the first year, the variance of the portfolio can be computed as:

$$(7)\ V_1 = \sum_{j=1}^{n} \sum_{i=1}^{n} dv_{i,j} x_{i,1} x_{j,1}$$

where
V_j is the variance of the portfolio for year j
$dv_{i,j}$ is the distributed covariance of projects i and j
$x_{i,j}$ is the decision variable associated to fund project i during year j,

For the second year, the variance of the portfolio can be computed as:

$$(8)\ V_2 = \left[x_1^T\right]\left[dv_2\right]\left[x_1\right] + \left[x_2^T\right]\left[\left(1+\text{int}\right)^2 dv_1\right]\left[x_2\right]$$

This equation accounts for the variance of the projects selected during the first year that are still ongoing on the second year, plus the variance of the projects selected to start on the second year adjusted by inflation.

In general, for any year j the total variance of the portfolio can be computed as:

$$(9)\ V_j = \sum_{i=1}^{j} \left[x_i^T\right]\left[\left(1+\text{int}\right)^{2(i-1)} dv_{j-i+1}\right]\left[x_i\right]$$

Equation (9) can also be written in a vector matrix from by creating an extended distributed variance matrix, similarly as done for the evaluation of the cost of the portfolio. The factor multiplying the original variance in this case is $\left(1+\text{int}\right)^{2(j-1)}$ for any given year j. An extended distribution variance matrix edv starts from the original dv matrix. Immediately below but offset by one column, we place the matrix $\left(1+\text{int}\right)^{2(j-1)} dv$ for each year into consideration.

With this extended distribution matrix, the variance of the portfolio can be computed for each year as:

$$(10)\ V_j = \left[x^T\right]\left[edv\right]\left[x_j\right]$$

where
V_j is the variance of the portfolio for year j

x^T is the transposed vector of the solution
edv is the extended distributed variance matrix
x_j is the decision variable corresponding to the year j

The extended distributed variance for the five projects presented before is as shown in Table 8.

Table 9 Extended distributed variance

edv	1	2	3	4	5
1	29,764,566,278	7,441,141,569	0	0	0
2	58,087,907,172	0	0	0	0
3	482,373,000,000	482,373,000,000	482,373,000,000	0	0
4	33,605,893,345	33,605,893,345	33,605,893,345	8,401,473,336	0
5	183,022,000,000	183,022,000,000	45,755,434,587	0	0
6	0	32,815,434,321	8,203,858,580	0	0
7	0	64,041,917,657	0	0	0
8	0	531,816,000,000	531,816,000,000	531,816,000,000	0
9	0	37,050,497,412	37,050,497,412	37,050,497,412	9,262,624,353
10	0	201,781,000,000	201,781,000,000	50,445,366,632	0
11	0	0	36,179,016,339	9,044,754,085	0
12	0	0	70,606,214,217	0	0
13	0	0	586,327,000,000	586,327,000,000	586,327,000,000
14	0	0	40,848,173,397	40,848,173,397	40,848,173,397
15	0	0	222,464,000,000	222,464,000,000	55,616,016,712
16	0	0	0	39,887,365,514	9,971,841,378
17	0	0	0	77,843,351,174	0
18	0	0	0	646,426,000,000	646,426,000,000
19	0	0	0	45,035,111,170	45,035,111,170
20	0	0	0	245,267,000,000	245,267,000,000

Similarly as with the extended cost matrix, the extended distributed variance matrix can be calculated until the 8th year, but we limited our calculations to the first five years.

3.4 Selection of the Portfolio

The decision of which projects to select can be found by solving an optimization problem. For our case, the objectives are two: maximize the value of the priorities and minimize total slack between the portfolio selection for the year and the budget. The second objective helps the agency not to loose unallocated funds at the end of the fiscal year.

$$\text{(11) Max: } \sum_{j=1}^{Y}\sum_{i=1}^{n} p_{ij}x_{ij}$$

$$\text{(12) Min: } \sum b - \left[edc^T\right]\left[x\right]$$

subject to:

(13) $P\left(\left[edc^T\right]\left[x\right] \le b\right) \ge \alpha$

(14) $\sum_i x_{i,j} \le 1, \forall j = \{1, 2, .., y\}$

(15) $x_{ij} \in \{0,1\}$

where

p_{ij} represents the benefit obtained by including project i in the solution of year j

x_{ij} is a binary decision variable to fund ($x_{i,j}$ =1), or not to fund ($x_{i,j}$ =0) project i during year j .

x is a column vector of the decision variables

edc^T is the transposed matrix of the extended distribution cost matrix.

b is a column vector with the maximum budget available during each year.

α is a column vector with the desired probability in each entry.

Constraint (13) ensures that the sum of the costs do not exceed the budget for each of the years under consideration. Constraint (14) ensures that each project is selected only once. Constraint (15) ensures that the project are completely funded or not funded at all.

3.5 Solution of the Problem

The fundamental difficulty encountered when trying to solve this problem is that constraint (13) is a probabilistic constraint. However, based on the simple project selection problem the constraint can be converted to a non-linear deterministic equivalent constraint (Charnes and Cooper 1959; Vajda 1972). Using a similar approach as the one by presented in Gabriel et al. (2006), but expanded for a multiperiod problem with inflation we can end with a deterministic equivalent constraint.

The single period portfolio optimization problem and the corresponding transformation of the probabilistic constraint to a non linear deterministic equivalent constraint as presented by Gabriel et. al 2006 can be extended to a multi-period optimization problem with inflation as follows:

For each of the years j in {1, 2,…,y} we have:

$$P\left(\left[edc^T\right]\left[x\right] \le b\right) \ge \alpha \Leftrightarrow$$

$$(16)\quad P\left[\frac{\left(\left[edc^T\right]\left[x\right] - \left[\overline{edc}^T\right]\left[x\right]\right)}{s} \le \frac{\left(b - \left[\overline{edc}^T\right]\left[x\right]\right)}{s}\right] \ge \alpha$$

where:

$$(17)\quad s = \left(V_j\right)^{1/2} = \left(\left[x^T\right]\left[edv\right]\left[x_j\right]\right)^{1/2}$$

is the standard deviation of the total cost for the portfolio of projects for year j. Since we consider the cost of each project to be normally distributed, then (16) can be written as:

$$(18)\ P\left(z \le \left(b_j - \left[\overline{edc}^T \right] [x] \right) \Big/ s \right) \ge \alpha$$

where z is a normal distribution with mean zero (0) and variance one (1).
This is by definition the cumulative distribution function, which means

$$(19)\ F\left[\left(b_j - \left[\overline{edc}^T \right] [x] \right) \Big/ s \right] \ge \alpha$$

Because this function is strictly increasing and invertible, the following equivalent constraint can be used:

$$(20)\ F^{-1}(\alpha)\left(\left[x^T \right] [edv] \left[x_j \right] \right)^{1/2} + \left[\overline{edc}^T \right] [x] \le b_j ; \forall j = \{1, 2, .., y\}$$

The two objectives (11) and (12) can be combined into one objective using a weight factor $0 < w < 1$ as follows:

$$(21)\ \text{Max: } w \sum_{j=1}^{Y} \sum_{i=1}^{n} p_{ij} x_{ij} - (1-w)\left(b - [edc][x] \right)$$

The model then becomes:

$$(21)\ \text{Max: } w \sum_{j=1}^{Y} \sum_{i=1}^{n} p_{ij} x_{ij} - (1-w)\left(b - \left[edc^T \right] [x] \right)$$

s.t.

(22)

$$F^{-1}(\alpha)\left(\left[x^T \right] [edv] \left[x_j \right] \right)^{1/2} + \left[\overline{edc}^T \right] [x] \le b_j ; \forall j = \{1, 2, .., y\}$$

$$(23)\ \sum_i x_{i,j} \le 1, \forall j = \{1, 2, .., y\}$$

$$(24)\ x_{ij} \in \{0, 1\}$$

If the confidence level α is selected to be greater or equal to 0.5 then constraint (22) is a convex quadratic constraint (Bazaraa et al. 1979) that is equivalent to the original probabilistic constraint (Gabriel et al. 2006) . The solution to the problem with strictly positive weights provides Pareto optimal points (Cohon 1978).

4. CASE STUDY

In the agency from which this example was drawn, three or more portfolio alternatives are developed in each study based upon the criteria of benefit vs. cost. There is a great magnitude of outside pressures that weigh upon each project (typically upper leadership bureaucracy). In a committee setting, the committee irons out which alternatives will be selected based upon subjective judgments of the criteria and need. Once a project alternative is selected by the committee it is placed in a queue for funding. A funding order is then selected based upon the budget per year. The process is not orderly and often spills over from meeting to meeting and into teleconferences and emails until someone's will is broken and or "the squeaky wheel gets the grease". Budget limitations are only one aspect of the problem, there are also benefits associated with each project and different stakeholders who are adamant about having their project funded, there are also uncertainties associated with the final costs of each project, inflation also factors in for the timing of the selection. Funding for projects is distributed by fiscal year and the funds for that year must be allocated or they will revert to another service or be rescinded. The funds cannot spill over into the following year or be used for prior years. Once the budget is allocated to the projects, it is sometimes possible to switch funds from one project to another however; this usually is only possible within the same funding year at the end of one of the projects, when the savings are realized. A typical occurrence in the process is to have a reserve of small projects called Unspecified Minor Construction (UMC) projects that can be used to allocate any excess funding to after the major project selections have been made. These projects range from $1.5 million dollars and below. They basically have no order of precedence and can be inserted into the funding stream as needed. For years, there has been no formal way of selecting projects other than the laborious conflict riddled process described above. The process often placed projects in order of funding based on large facility need and not necessarily what was best for the program's portfolio. This process would please large facility leadership and enrage leadership in the smaller facilities due to the bureaucratic pull of the larger facilities. A more organized process is required where the subjective voting on criteria can be done without political influence and a weighted decision established that equates to a more mathematically controlled and fair process.

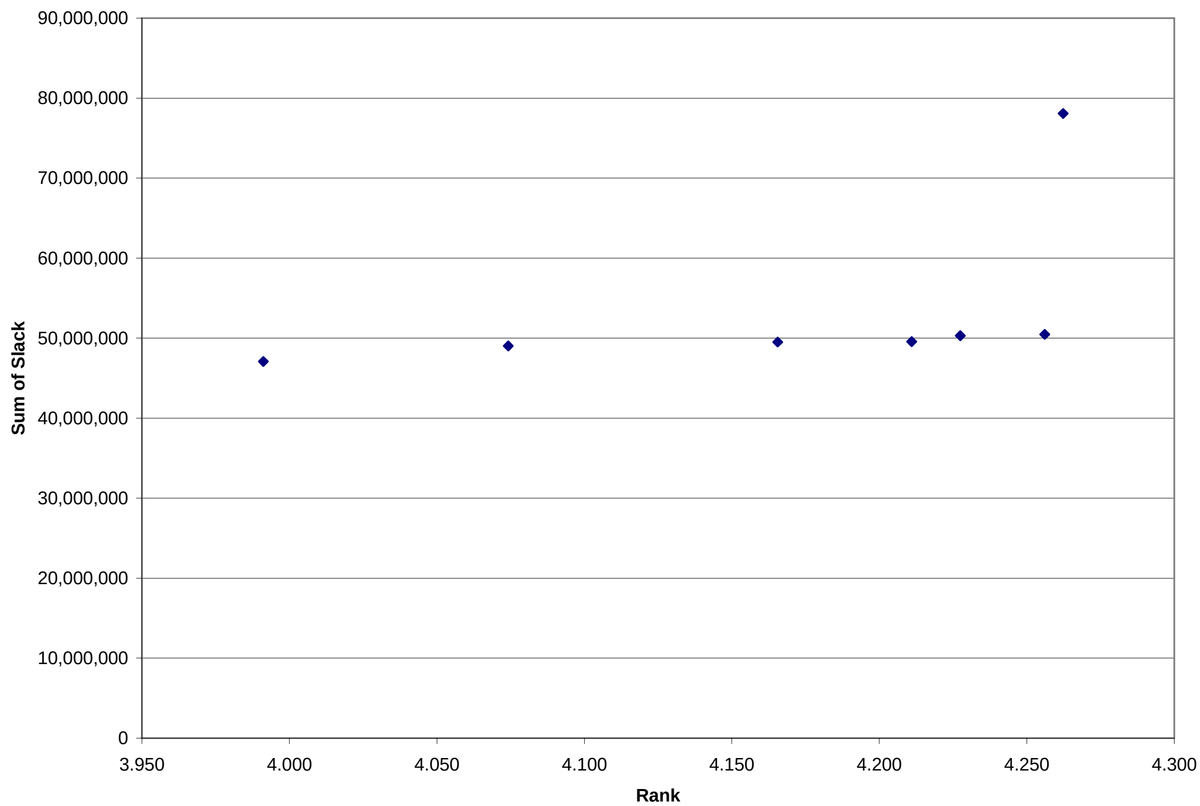
To illustrate the procedure we have chosen a set of five projects with durations, cost and variances as presented before. The solution to this problem depends on the weight w used and the confidence level α selected. We have obtained the Pareto optimal points using an exhaustive search, and a confidence level of 95%. The Pareto optimal results are presented below in Table 10.

Table 10 Pareto Optimal Selection of Portfolios

	Year 1	Year 2	Year 3	Year 4	Year 5	Rank	Slack
Portfolio 1	3	4		1,5	2	3.991	$47,086,956
Portfolio 2	3,4			1,5	2	4.074	$49,027,975
Portfolio 3	1,5	2	3	4		4.166	$49,524,296
Portfolio 4	1,3	4		5	2	4.211	$49,554,985
Portfolio 5	1,2,5		3	4		4.227	$50,307,176
Portfolio 6	1,3	4		2,5		4.256	$50,461,266
Portfolio 7	1,4,5	2		3		4.262	$78,079,799

We noted that almost all years the probability to remain under the budget was higher than 98% in all cases, except for Portfolio 6 which has a 96.8% of confidence level in year 2. Some interesting observations can be made from Table 9. We notice that project 1 was selected by 5 of the 7 Pareto optimal points to start in year 1, 6 of the 7 solutions selected Project 3 to start either on year 1 or year 3. Surprisingly, the solution with the smallest total slack has a very high probability (close to 1) in all years to remain under budget. We note how Portfolio 7 would probably be discarded by the analyst because although has the highest rank of all, the difference in total slack is very high, it is 20 millions over Portfolio 6 which has a rank very close to Portfolio 7. We can see the difference graphically in Figure 1.

Figure 1 Pareto Optimal Set



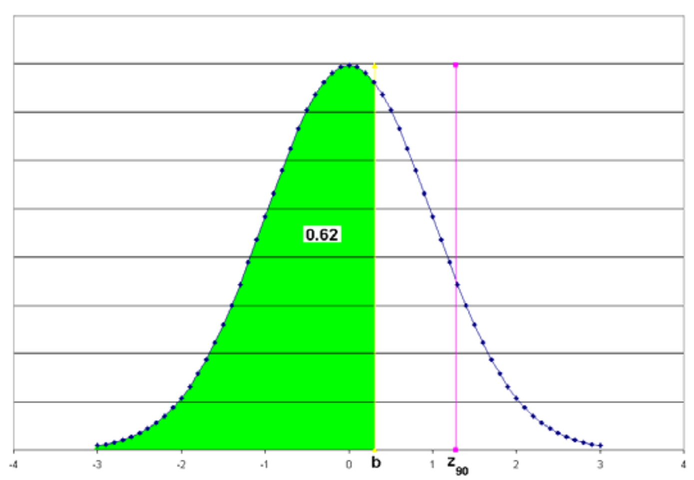
An interesting portfolio is shown in Table 10. This portfolio selection has a Rank 4.267 which is larger than the rank of any of the previous solutions, and a total slack of around 56 millions which is not too large. This option seems good at first look, since the values of each year are below the budget and the rank is very high. However, when the probabilities are analyzed, one immediately notices that the probability to remain under budget for year 1 is only 62%. If this portfolio is selected, there is a 38% chance of going over budget during the first year.

Table 11 Dangerous portfolio selection that seems like a good choice

	Year 1	Year 2	Year 3	Year 4	Year 5	Rank	Slack
Projects	2,3	1	4	5		4.267	$56,222,556
Exp. Cost	$57,679,470	$52,982,191	$59,730,453	$42,192,664	$42,192,664		
Budget	$58,000,000	$60,000,000	$64,000,000	$64,000,000	$65,000,000		
P(Cost<=B)	0.62	1.00	1.00	1.00	1.00		

This situation can be graphically explained by looking at the normal distribution as presented in Figure 2.

Figure 2 Normal Distribution for the portfolio selected in Table 10



The center of the normal distribution is the expected cost of the portfolio, the value b is the year 1 budget and the value z_{90} represents the ideal budget value at which the probability of being under budget is 90%. Clearly the selection of the confidence level α is critical in finding portfolio options that are optimal.

5. CONCLUSIONS

Portfolio selection under uncertainty is a challenge for many organizations, which can be solved using optimization models with non linear constraints. The model presented gives the decision makers the flexibility required to assign different ranks to the projects if they are funded in different years while accounts for the effect of inflation which increases the cost of the projects as the time passes.

A portfolio selection that is below the budget for every year not necessarily is a good solution. It is possible to have a probability of running over budget larger than the acceptable risk. Using a model that accounts for the multi-year aspects of the problem and includes constraints to ensure that the risk of going overbudget is maintain under limits, the decision makers can choose one solution among the Pareto optimal solutions provided.

Changing the confidence level will result in a different set of Pareto optimal solutions, as the confidence level is dropped down, solutions that have a higher risk of going overbudget will be included in the set. This confidence level is then a very important parameter in the solution of the problem.

6. REFERENCES

Alidi, A.S. (1996) “Use of the analytic hierarchy process to measure the initial viability of industrial projects”. International Journal of Project Management Vol. 14, No. 4, pp. 205-208.

Amos, Scott (2004) “Skills & Knowledge of Cost Engineering, 5th Edition, Revised”. AACE International Morgantown, WV.

Argote, Linda (1982). “Input Uncertainty and Organizational Coordination in Hospital Emergency Units”. Administrative Science Quarterly, Vol. 27, No. 3., pp. 420-434.

Badiru, A.B., Pulat, P.S. (1995) “Comprehensive Project Management: Integrating Optimization Models, Management Principles and Computers”

Cheng, E.W.L., Li, H. (2001) “Information priority-setting for better resource allocation using analytical hierarchy process (AHP)”. Journal of Information Management and Computer Security, 9/2, pp. 61-70.

Cheng, E.W.L., Li, H. (2005) “Analytic Network Process Applied to Project Selection”. Journal of Construction Engineering and Management, 0733-9364, 131:4(459).
Detemple, Jérôme B., Garcia, René, Rindisbacher, Marcel (2003). “A Monte Carlo Method for Optimal Portfolios” The Journal of Finance, Vol. 58, No. 1., pp. 401-446.
Gabriel, S. A., Kumar, S., Ordóñez, J., and Nasserian, A. (2006a). "A Multiobjective Optimization Model for Project Selection with Probabilistic Considerations," Socio-Economic Planning Sciences, 40, 397-313.
Gabriel, S., Ordonez, J., and Faria, J. (2005) “Contingency Planning in Project Selection Using Multiobjective Optimization and Chance Constraints”. Journal of Infrastructure Systems, 1076-0342, 12:2(112).
Gabriel, S.A., Ordóñez, J.F., and Faria, J. A.. (2006). “Contingency Planning in Project Selection Using Multiobjective Optimization and Chance Constraints,” ASCE Journal of Infrastructure Systems, Vol. 12, No. 2, pp. 112-120.
Garvey, Paul R. (2000) “Probability Methods for Cost Uncertainty Analysis” marcel Dekker, Inc. New York, NY.
Gori, E. (1996) “Portfolio selection of capital investment projects in the Durban Metropolitan Region”. Journal of Construction Management and Economics, Vol. 14, No. 5, pp. 451-456.
Kujawski, Edouard (2002). “Selection of Technical Risk Responses for Efficient Contingencies”. Systems Engineering Vol. 5, No. 3, pp 194-212.
Li, Z., Puyan, M. (2006) “A Stochastic Optimization Model for Highway Project Selection and Programming under Budget Uncertainty”. Applications of Advanced Technology in Transportation 9th International Conference.
Lockett, A. G. and Freeman, P. (1970). “Probabilistic Networks and R & D Portfolio Selection”. Operational Research Quarterly, Vol. 21, No. 3. pp. 353-359.
Malcolm, D. G., Roseboom, J. H., Clark, C. E., and Fazar W. (1959) “Application of a Technique for Research and Development Program Evaluation”. Operations Research, Vol. 7, No. 5. pp. 646-669.
Mandakovic, Tomislav and Souder, William E. (1985). “An interactive decomposable heuristic for project selection.” Management Science, Vol. 31, No. 10, pp. 1257-1271.
Markowitz, Harry (1952). “Portfolio Selection”. The Journal of Finance. Vol. 7, No. 1, pp 77-91.
Martino, Joseph Paul (1995). “Research and Development Project Selection”. Wiley-Interscience,
Mohanty, R. P. (1992) “Project selection by a multiple-criteria decision-making method: An example from a developing country”. International Journal of Project Management. Vol. 10, No. 1, pp. 31-38.
Moselhi, O., Deb, B. (1993) “Project selection considering risk”. Journal of Construction Management and Economics, Vol. 11, No. 1, pp. 45-52.
Mustafa, M.A.; Al-Bahar, J.F. (1991) “Project risk assessment using the analytic hierarchy process”. Journal of Engineering Management, Vol. 38, No. 1, pp. 46 – 52.
Saaty, T. (1980) “The Analytic Hierarchy Process”. New York: Mc Graw Hill. USA
Sinuany-Stern, Zilla and Mehrez, Abraham (1987). “Discrete Multiattribute Utility Approach to Project Selection”. The Journal of the Operational Research Society, Vol. 38, No. 12. pp. 1133-1139.
Steuer, R.E. (1986) “Multiple Criteria Optimization: Theory computation and application”. Wiley. New York, NY.
Stewart, T.J. (1991) “A Multi-Criteria Decision Support System for R&D Project Selection”. The Journal of the Operational Research Society, Vol. 42, No. 1, pp. 17-26.
Stewart, Theodor J. (1991) “A Multi-Criteria Decision Support System for R&D Project Selection”. The Journal of the Operational Research Society, Vol. 42, No. 1., pp. 17-26.
Taylor, Bernard W. III, Moore, Laurence J., and Clayton, Edward R. (1982). “R & D Project Selection and Manpower Allocation with Integer Nonlinear Goal Programming”. Management Science, Vol. 28, No. 10., pp. 1149-1158.
Williams, D. J. (1969). “A Study of a Decision Model for R & D Project Selection”. Operations Research, Vol. 20, No. 3. pp. 361-373.
Winston, Wayne L. (2004) “Operations Research: Applications and Algorithms. Fourth Edition”. Brooks Cole Belmont, CA.