

## **Using a Multiobjective Local Search Procedure for Construction Time-Cost Analysis**

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### **Abstract**

The time-cost trade-off problem, i.e. selecting appropriate resources to perform the activities of a project so that to balance the project total duration and the project total cost, is one of the most important aspects of construction and business projects planning. The literature for the case where the time-cost relationships are defined at discrete points, known as the discrete time-cost trade-off problem, has been rather sparse. In any case, it seems that new generation multiobjective techniques have not used to solving it. In this paper we propose a model of project scheduling with two conflicting objectives, i.e. minimize the total direct project cost and minimize the project duration, for the discrete time-cost trade-off problem. We apply a recently developed multiobjective evolutionary algorithm, the so-called the Pareto Archived Evolution Strategy (PAES), for approximating the Pareto front of the non-dominated solutions which employs local search and uses a reference archive of previously found solutions in order to identify the approximate dominance ranking of the current and candidate solutions. We also discuss some issues relating to its implementation.

### **Keywords**

Project scheduling, Time-cost trade-off problem, Multi-objective optimization, PAES

## **1. Introduction**

The importance of the time-cost trade-off problem was recognized over forty years ago, since the initial development of project scheduling techniques (Fulkerson, 1961, Kelly, 1961). Given that the duration of a specific activity is reduced but its direct cost is increased, as additional resources are required for its execution, decisions must be made on the carrying out of activities in order to balance the project total duration and the project total cost. This problem, known in the literature as the “time-cost trade-off” problem, has become a central issue in construction planning and control (Liberatore et al., 2001, Shtub et al., 1994).

Whilst time-cost trade-off problems have been extensively studied in the case of continuous time-cost relationships, little has been achieved in the more realistic case of the discrete time-cost trade-off problem. The discrete time-cost trade-off problem is a hard combinatorial problem, in the sense that exact procedures can only resolve problems of small dimensions (De et al. (1995), Demeulemeester et al. (1996)). Consequently, efforts have been focused on the development of heuristic and metaheuristic algorithms (Anagnostopoulos and Kotsikas, 2001, De et al., 1995, Feng et al., 1997, Li and Love, 1997).

This paper is dealing with project scheduling with two conflicting objectives simultaneously, namely, minimize the direct cost of the project, and minimize the total duration of the project, when discrete time-cost relationships are allowed on project activities. A recently developed multiobjective evolutionary algorithm (Knowles and Corne 2000), called the Pareto Archived Evolution Strategy (PAES), which is based on local search to generate new candidate solutions, the multiobjective analogue of a hillclimber, and on population information to evaluate the quality of solutions, is proposed for constructing the Pareto front of non-dominated solutions. We also discuss issues concerning the implementation of the algorithm in order to improve its time efficiency in the time-robustness problem.

Various multiobjective algorithms have been proposed over recent years that solve multiobjective optimization problems without converting the problem to a single-objective one (Coello, 2000). However, the number of applications of these techniques to engineering problems, in particular to project scheduling, is still scarce. In Anagnostopoulos and Kotsikas (2004) the time-robustness problem is resolved using the Pareto Simulated Annealing multiobjective algorithm and Zheng et al. (2004) use a genetic algorithm-based multiobjective procedure based on an adaptive weight approach for solving the discrete time-cost trade-off problem.

## 2. The discrete trade-off problem

Assuming that  $G = (N, A)$  is the activity-on-nodes network of a project, where  $N$  is the set of  $n$  nodes representing activities and  $A$  the set of  $m$  arcs, representing the precedence relations. An activity may be performed by various discrete time-cost combinations. The direct cost is a non-increasing function of time. Let  $K(i)$  denote the set of all feasible time-cost combinations  $(d(i,k), c(i,k))$  for the activity  $i$ , where  $d(i,k)$  is the duration and  $c(i,k)$  is the (direct) cost of the activity  $i$  when it is performed with the combination  $k$ . Assuming that time-cost combinations for each set  $K(i)$  are ranked in increased order of magnitude as for the duration  $d(i,k)$ , i.e. if  $k$  and  $l$  are combinations such that  $k < l$  then  $d(i,k) < d(i,l)$  and  $c(i,k) > c(i,l)$ .

A feasible solution (or project schedule)  $s$  is the network  $G$  containing an allowable duration in each of its activities. Since the precedence constraints between activities are determined by the structure of the network and each activity has an allowable duration, we may calculate the early and late time of each event (node). Therefore the project scheduling can be determined. The network schedule has a total duration equal to the longest path from node 1 to  $n$ , and total cost equal to

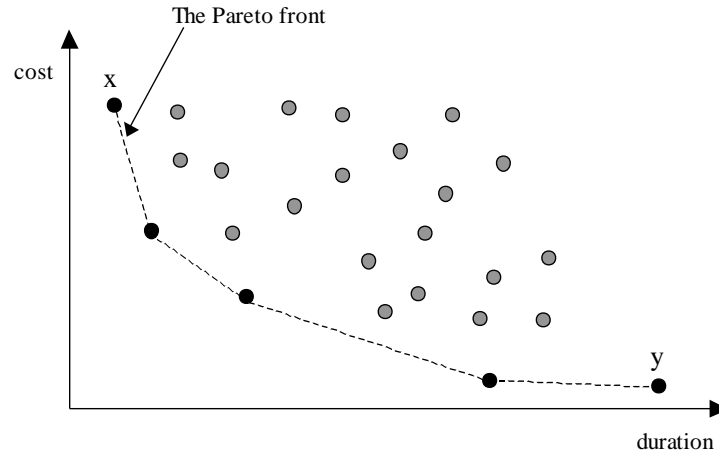
$$\sum c(i,k), \text{ for each activity } i \text{ performed with the combination } k \text{ from } K(i)$$

The maximum and minimum duration schedules are the two extreme cases. The network, with the maximum (minimum) selected duration for each activity, has the minimum (maximum) total direct cost and the longest (minimum) total project duration (point y (point x), fig. 1).

The objective is to find the project time-cost trade-off curve, i.e., the efficient time-cost curve showing the relationship between project duration and cost over the feasible solutions (fig. 1). The trade-off curve is the so called Pareto front (or the non-dominated set), i.e., the set of solutions such that no other feasible solutions exist that have better objective values in both time and cost than the solutions in the non-

dominated set. Given that the time-cost combinations of each activity are discrete, the time-cost curve is not convex.

This is a biobjective optimization problem in which we search to minimize at the same time the project duration and the project total cost, in the case that only discrete time-cost combinations are allowed on the project activities. Unfortunately, these criteria are conflicting to each other, since the duration of a specific activity is reduced but its direct cost is increased, as additional resources are required for its execution.



**Figure 1: Time-cost trade-off curve of a project.**

### 3. The Pareto Archived Evolution Strategy (PAES)

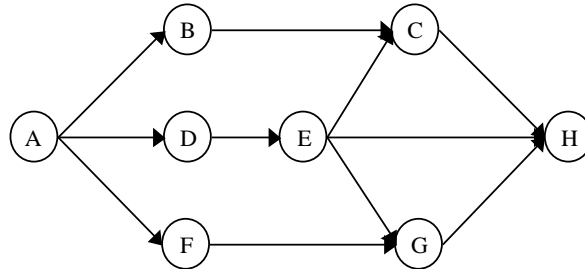
Knowles and Corne (2000) have proposed a simple and fast evolutionary algorithm called the Pareto Archived Evolution Strategy (PAES) for finding the set of nondominated solutions. In PAES from the current solution a new solution is generated, namely the mutated solution. The mutated solution is compared with the current. If the mutated solution dominates the current solution, i.e. the mutated solution has smaller duration and smaller cost of the current solution, the mutated solution is accepted as the next current solution and the iteration continues. If the current solution dominates the mutated, the latter is discarded and a new mutated solution is generated. For maintaining population diversity along Pareto front, an external archive  $Q$  of nondominated solutions is considered. If the mutated and the current solutions do not dominate each other, the new mutated solution is compared with each solution of  $Q$  to verify if it dominates any member of the set. If so, then the mutated is added in  $Q$  and becomes the next current. The dominated solutions are eliminated from  $Q$ . If the mutated does not dominate any member of the set, both current and mutated are checked for their nearness with the solutions of the set. If the mutated resides in the least crowded region in the parameter space among the members of the archive, it is accepted as the next current and is added to  $Q$ . The procedure is terminated if a condition based on the archive or number of iterations is satisfied.

Note that, though the archive can be seen as the population analogue of evolutionary algorithms, PAES is an authentic local search procedure, because it uses a single solution local search and replaces it with only one solution.

2.

In order to cut down the computational cost of the algorithm, PAES uses a crowding procedure based on recursively dividing up the 2-dimensional objective space. The grid location in objective space is determined for each generated solution. The number of solutions residing in a grid location is called its population. The required grid location can be found by repeatedly bisecting the range in each objective and finding in which half the solution lies, given that the range of the space is defined in each objective. The solutions in each grid location, its population, are maintained. The recursive subdivision of the space and

assignment of grid locations is carried out using an adaptive method that works by calculating the range in objective space of the current solutions in the archive and adjusting the grid so that it covers this range. Grid locations are then recalculated. To avoid recalculating the ranges too frequently, the subdivision of the space is done only when the range of the objective space of archived solutions changes by a threshold amount. The number of divisions of the space required is the only parameter that must be set.



**Figure 2: The Activities on Nodes network of the test project.**

### 3.1 A PAES algorithm for the time-cost trade-off problem

A crucial point for the performance of PAES algorithm is the definition of the mutation procedure that will be used. In our algorithm the mutation procedure is defined as follows. In each activity we assign a counter, which counts how many times an activity was critical. When a new solution is generated, we check which activity is critical. If an activity is critical, its counter is increased by one. To mutate the current solution, an activity is randomly selected according to its probability to be critical. The time-cost combination of this activity is replaced by a different randomly selected time-cost combination. An activity with low probability to be critical is also selected and, if its time-cost combination is  $k$  another, combination  $l$  is selected such that  $d(i,l) > d(i,k)$  and  $c(i,l) < c(i,k)$ . By this procedure we hope that we will find solutions with different duration and cost as low as possible.

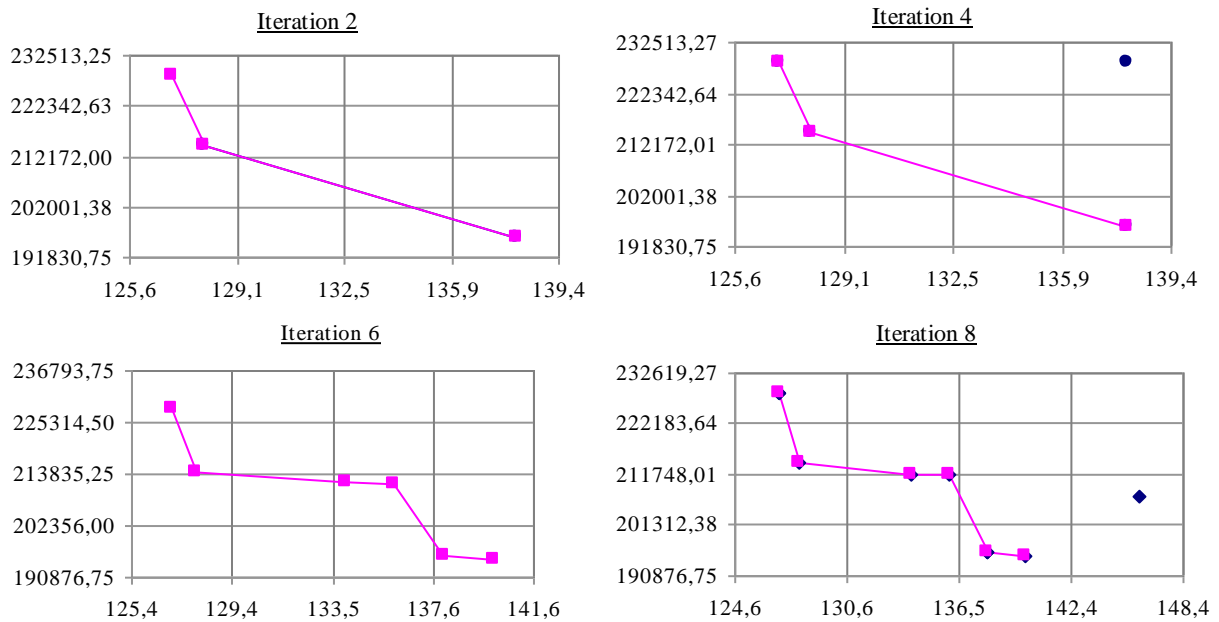
In order to improve the performance of the algorithm for the time-cost trade-off problem when a predefined number of iterations have been executed, we optimize each solution in the archive. If the current combination of a non-critical activity  $i$  is  $k$ , we select a new combination  $l$  which minimizes the difference  $TF(i) - [d(i,k) - d(i,l)]$ , where  $TF(i)$  is the total float of activity  $i$ , and this difference is positive. This procedure is repeated for each non-critical activity of all solutions. By this procedure, the direct cost of each solution of the archive is reduced as much as possible but while its duration remains unchanged. The performances of the PAES depend of the values of its parameters. The number of subdivisions of the objective space (it is proposed that this number should be equal to  $2^p$  where  $p$  is an integer to be determined), the size of the archive  $Q$ , the stopping criteria of the algorithm and how many times the archive set should be optimized. The appropriate values of those parameters will be set when a computational analysis with a full factorial design will be performed. To illustrate the grid and the Pareto-front, the standard PAES has applied to a randomly constructed project with eight activities, using a first version of its code (Fig.1). For each activity five time-cost combinations were available. The number of subdivisions was  $2^2 = 4$ . The algorithm has applied eight times to the project. Figure 2 shows the evolution of the grid and of the Pareto-front.

## 4. Conclusions

This paper proposes a procedure that aims to solve the time-cost trade-off problem in the case that discrete time-cost relationships are allowed on project activities. This is a typical multiobjective problem with two

conflicting objectives. In order to construct the Pareto set of optimal solutions, we use a recently developed multiobjective evolutionary procedure, the Pareto Archived Evolution Strategy. This simple and fast algorithm is based on local search to generate new candidate solutions and on archived nondominated solutions to evaluate the quality of solutions. We have also make suggestions concerning the implementation of the algorithm in order to improve its effectiveness.

Our research is currently directed towards two objectives. On the one hand, for testing its performances, the algorithm is coded using the VBA on the Microsoft *Project* platform. In a first stage will try to find the optimal values of the algorithm parameters and, in a second stage, to test the time efficiency of the algorithm as well as the quality of the obtained solutions. On the other hand, we adapt the PAES algorithm for solving the so-called discrete time-robustness trade-off problem, in which the robustness is defined as a weighted sum of total slack of project activities.



**Figure 3: The evolution of the grid and of the Pareto front.**

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