A Multi Objective Linear Programming Model for Scheduling of Linear Repetitive Projects

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Abstract

The Critical Path Method (CPM) and the Repetitive Scheduling Method (RSM) are the most often used tools for the planning, scheduling and control Linear Repetitive Projects (LRPs). CPM focuses mostly on project's duration and critical activities, while RSM focuses on resource continuity. In this paper we present a linear programming approach to address the multi objective nature of decisions construction managers face in scheduling LRPs. The Multi Objective Linear Programming model (MOLP-LRP) is a parametric model that can optimize a schedule in terms of duration, work-breaks, unit completion time and respective costs, while at the same time the LP range analysis can provide useful information regarding cost tradeoffs between delay, work-break and unit delivery costs. Results are demonstrated with the use of a standard LRP literature example.

Keywords

Linear Projects, Scheduling, Linear Programming

1. Introduction

Linear Repetitive construction Projects (LRPs) consist of a set of activities that are repeated sequentially at different locations or units (construction sites). The activities follow a logical and technological driven sequence described by time or distance constraints for the entire life span of the project (Kallantzis and Lambropoulos, 2004). During the last years different methods have been proposed for planning, scheduling and controlling the construction process in LRPs.

Since the seventies various researchers have been challenged the applicability of CPM in an attempt to prove its inadequacies in scheduling LRPs (Peer, 1974; to Harris and Ioannou 1998). The insufficiency of network analysis to describe the repetitive nature of the construction process, its weakness to provide uninterrupted utilization (work continuity) of resources and minimization of the idle time, and also the weakness to model the learning effect from the work repetition are stated as the main reasons. Moreover, the arbitrary estimation of activities' duration instead of the estimation of production rate, the large number of activities that are required in the network for large and complicate projects and the lack of information contained in the diagram are also considered significant weaknesses of CPM (Birrell G.S. 1980).

The Repetitive Scheduling method (RSM) that was introduced by Harris and Ioannou (1998), and was further developed with work that followed (Harris, Yang and Ioannou, 2001 to 2004) is pictured by a graphical representation of the project on an X-Y diagram where the repeated units (work progress) and the

elapsed project time are drawn on the two axes. The objective in RSM is not the minimization of the project completion time but achieving work continuity which leads to minimizing the overall project cost. The RSM's algorithm involves two stages: The first stage is similar to the forward pass computations of CPM and results in the computation minimum project duration. In the second stage each continuity relationship 'pulls' the predecessors to eliminate the time gap with the successor to ensure work continuity and uninterrupted resource utilization, in contrast with the CPM's push-system, where the start of every activity is pushed in time to maintain the precedence relationships with its predecessors.

Although RSM optimizes for continuity its analysis features are limited. It can only allow trade offs between time gaps and project duration on a trial and error basis. Cost considerations and other control variables are not taken directly into consideration in the computation, but only as back-end calculations.

Multiple Objective Scheduling Decisions in LRPs
Scheduling of LRPs is in practice more complicated and relevant decisions by construction managers could involve more control variables than just duration (CPM) and resource continuity (RSM) such as:
i.   **Duration**: Project duration is a key variable to any project.
ii. **Resource delay (RD)**: Violating the continuity of the same task between successive project units introduces work-gaps that increase the cost of the project because of idle resources.
iii. **Unit completion time (UCT)**: Completing the work on a project unit affects the project deliverables and it could affect the financial costs since project's cash receipts depend on deliverables.
iv. **Slack time (ST)**: Reducing activity slack time introduces higher risk to the project, regarding unit completion time and overall project duration.
v.   **Number of project's units**: Fixed cost associated with maintaining a construction site could increase the cost of the project but at the same time may improve the project's cash flow.

Moreover, scheduling decisions are rarely based only on any single variable. Alternative project schedules, comparisons and cost tradeoffs are often needed to arrive at an acceptable or optimum project schedule.
To address these issues, the following multi objective linear programming model (MOLP) for LRPs is introduced:

2. A Multi Objective LP model for Linear Repetitive Project Scheduling (MOLP - LRPS)

A formal description of the MOLP – LRP model
Any LRP can be defined by a set of **M** tasks and **P** project dependency relationships (SS, FS, SF, FF, with or without time-lag). The project is divided into **N** separate units in a linear way so that in general: a) All tasks are performed in all units, b) A task cannot be performed in any project unit before the same task is completed in the previous unit and c) The set of dependencies remain the same in all units. Yang, (2002a) lists a set of practical concerns in scheduling repetitive projects that are exceptions to these general assumptions, which however can be easily handled in the LP formulation that follows.

Model variables and parameters
Let $i= 1,,2,…,M$ denote the project tasks and $j=1,2,…,N$ the project units.
Define: d_{ij} the duration of task i in unit j *(alternatively it can be formulated as the amount of the corresponding work w_{ij} divided by the production rate p_{ij})*

s_{ij} , f_{ij} , the start and finish time respectively of task i in unit j
P_i the set of predecessor activities to task i
E the set of all activities without successors
WB_i the total time of work-breaks for task **i** because of discontinuities in successive units
UC_j the completion time of project unit **j**   and D_j the set delivery time of unit **j**

c_j the cost per time unit (penalty or financial) for delays in finishing unit **j**.

f_i the cost per time unit of work-break (idle resources) in task **i**.

Constraint definitions

The following set of constraints describes the operation of activities in an LRP:

a. Task duration constraints $f_{ij} = s_{ij} + d_{ij}$ $\forall$ *i=1,2,...,M,   j=1,2,...,N*

b. Project linearity constraints $s_{ij+1} \geq s_{ij}$ $\forall$ *i=1,2,...,M,   j=1,2,...,N-1*

(task in unit j follows the same task in unit j-1. Exceptions to this rule can be handled accordingly))

c. Time and distance dependencies $s_{ij} \geq f_{kj}$ $\forall$ *i=1,2,...,M,   j=1,2,...,N*

(the exact form of the constraint depends on the type of the dependency. Without loss of generality here we assume that all dependencies are FS type. Exceptions can be handled accordingly))

d. Unit completion time: $UC_j \geq f_{kj}$ $\forall$ *k* $\in$ *E*

(completion time for unit j, (UC_n equal the project's duration)

$$WB_i = \sum_{j=1}^{N-1}(s_{ij+1} - f_{ij}), \quad \forall \; i = 1,...,M \qquad WB = \sum_{i=1}^{M}WB_i$$

e. Resource delay:

(the sum of time gaps for task i, and the total resource time lost in work breaks delay)

Global Objective function

Depending on the values of the parameters c_j and f_i the following objective function:

$$\text{Minimize} \sum_{j=1}^{N}c_j.(UC_j - D_j) + \sum_{i=1}^{M}f_i.WB_i$$

can be used accordingly for optimizing:

i. Project Duration: Minimize UC_N *c_N=1, rest of c_j and f_i equal 0*

ii. Total work-break time: Minimize WB   *All f_i equal 1, all c_j equal to 0.*

iii. Unit Completion Time   Minimize $\sum_{i=1}^{M}UC_i$ *All f_i equal 0, all c_j equal to 1*

iv. Total Cost of work-break   Minimize $\sum_{i=1}^{M}f_i.WB_i$ *All c_j equal to 0.*

v. Cost of Delays in Unit Completion   Minimize $\sum_{i=1}^{M/}c_j.(UC_j - D_j)$ *All f_i equal 0.*

vi. Tradeoffs between Cost of Project delays and Resource Delays

$$\text{Minimize} \sum_{j=1}^{N}c_j.(UC_j - D_j) + \sum_{i=1}^{M}f_i.WB_i$$

3. MOLP – LRP Applications – Analysis of a case study

3.1 LRP - Case study example

In this section we demonstrate the type of answers and analysis that can be supported by the proposed MOLP – LRPS model through a specific example that was initially used by the RSM authors (Harris & Ioannou, 1998), which is used here, with some modifications to task duration times. The project consists of six repetitive units, each having six discrete activities repeated at each unit. Task dependencies are finish-to-start and each activity is performed by a specific crew. Figure 1 (left) shows the precedence network for one unit as well as the duration of each task at each of the six project units.

3.2 Results under different single-objective settings

3.2.1 Minimization of work-breaks under CPM duration

The CPM schedule for the project produces a minimum duration of 48 days. Given this duration as a constraint, the LP model was run for minimizing work-breaks (2.2.3.ii). The resulting schedule is shown in figure 1 (right). The minimum project duration of 48 days can be satisfied with a minimum of 24 days of work-breaks at tasks B and C. Further reduction of work-break time at activities cannot be achieved without extending the project's duration beyond 48 days.
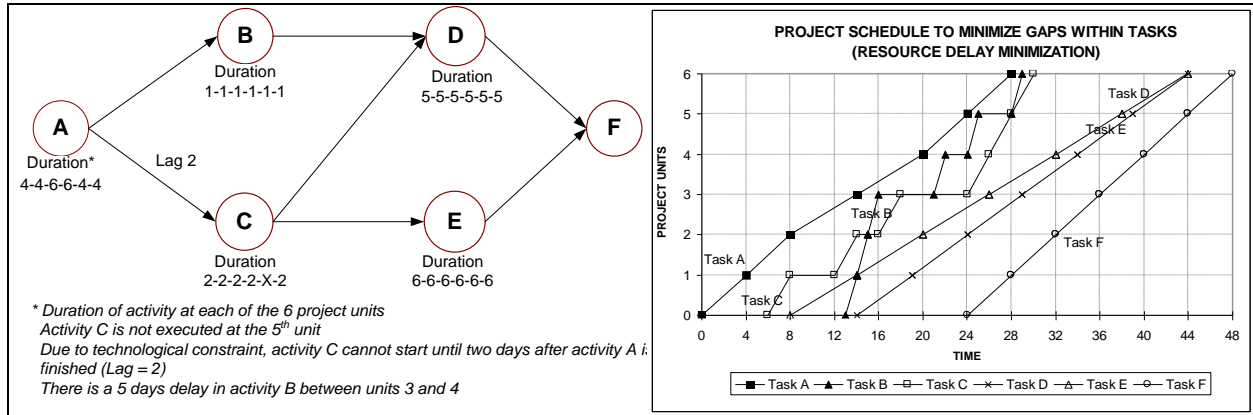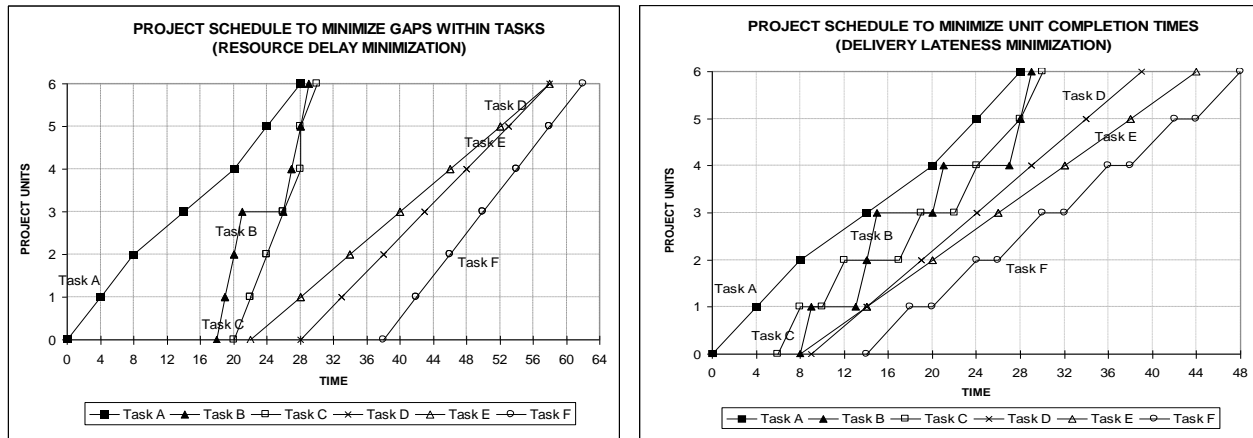

Figure 1: Precedence network and Schedule for CPM duration and minimum work-breaks

3.2.2 Minimization of work-breaks without the CPM duration constraint

If the CPM duration constraint is relaxed the work-breaks can be further reduced to a total of 5 days but this will result in extending the project's duration to 62 days as shown in figure 2 (left). The work break of task C is eliminated, while this of task B is reduced to 5 days as set by the project's technological constraints. The finishing time of all units is also pulled to 14 days later than in the previous schedule.

Project Duration = 62 days, *Work-breaks = 5 days*
 Unit Completion Times:
  1-42 days, 2-46d, 3-50d, 4-54d, 5-58d, 6-62d

Project Duration = 48 days, *Work-breaks = 63 days*
 Unit Completion Times: (average 19 days earlier)
  1-18 days, 2-24d, 3-30d, 4-36d, 5-42d, 6-48d

Figure 2. Minimization of Resource Delays / Minimization of Unit Completion Times

3.2.3 Minimizing unit completion time

If concern is focused in completing each unit as early as possible, additional work-breaks must be inserted at tasks. Figure 2 (right) shows the resulting schedule when unit completion time is sought as the objective. Still the overall duration of the project cannot be further reduced to less than 48 days but intermediate unit are delivered at earlier times as shown (in parenthesis the time saved in delivering each unit as compared to the schedule in figure1). The total work-break time is increased to 68 days, 63 more than the minimum and 44 days more than the same duration CPM schedule (figure 1) , for expediting the delivery of units by an average of 19 days.

3.3 Cost trade-offs between work-break and unit duration

The cost of work-breaks at different activities varies according to the type and scarceness of the resources consumed. The same is true with the completion time of different units which can affect the cost of the project either directly (i.e. delay penalties) or indirectly (i.e. financial cost due to late cash receipts). The cost objective functions (2.2.3 iv, v and vi) can be used for minimizing a certain type of cost or a trade-off analysis between different types of costs.
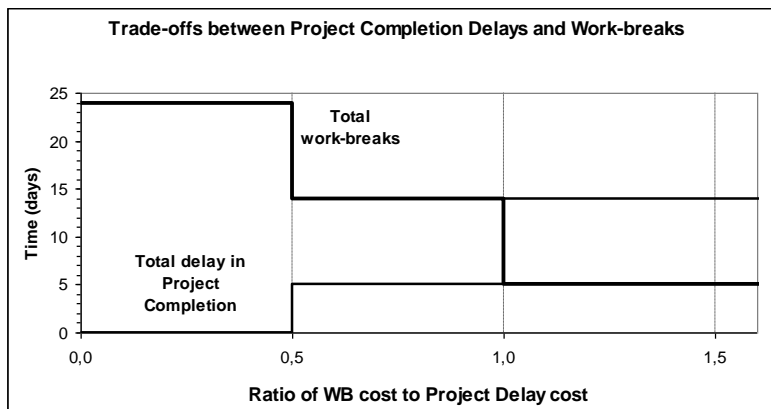Even in the case where no exact cost data exist, the standard tools of LP range and sensitivity analysis can be used to establish optimum scheduling at different relative cost relationships. The following two examples demonstrate this type of analysis.

3.3.1 Trade off between project duration delay cost and work-break cost
Delays are measured as deviations from the earliest finish date of the project as set by the CPM. It is also assumed that intermediate delays in completing each unit do not impose any cost to the project, and that the cost of work-breaks is the same for all tasks.
In this case the objective function of the LP model is set to: *Minimize UC_n+WB*.
Since only the relationship between the two types of cost and not their absolute values affect the optimization process, both values are initially set equal to one. LP range analysis defines the optimality ranges and the changes in the optimum solution can be derived. The results are shown in figure 3.

- For work-break cost up to 50% of the cost for project delays, the optimum scheduling has duration of 48 days (minimum possible) with maximum work-break time of 24 days.
- When work-break cost exceeds 50% of the project delay cost, it is more economical to let the project slip by 5 days in order to reduce work-breaks to 14 days.
- When the work-break cost exceeds the project delay cost the optimum schedule is the one that introduces the minimum of 5 days of work-breaks, which results in extending the project duration by 14 days.

Figure 3. Tradeoffs between project completion and work-breaks.

3.3.2 Trade off between unit delay cost and work-break cost
In the second example it is assumed that delay costs are paid not only when the total project finish time slips but when delays also exists in delivery time of each project unit. Delays are measured as deviations

from the earliest finish dates of the project units as they are set by the CPM and have the same cost across all units. The cost of work-breaks is assumed to be the same for all tasks as in the previous example. In this case the objective function of the LP model is set to:

$$Minimize \sum_{j=1}^{n}(UC_j - MinUC_j) + WB$$

The range analysis results are shown in figure 5. Minimizing the work break time in the task is necessary only when the associated work-break cost is 6 times higher than the cost paid for unit delays 4. A significant break point is when work-break cost is 3 times higher than cost of delays. Under this level total delays in the units are kept below 17 days in total (about 3 days per unit) while above this level, they range from 51 to 114 days (about 8,5 to 19 days). Figure 4 gives a graphical representation of the results.

Ratio of Work-break cost to Unit delay cost (f_i / c_i)	Total delay in all units*	Total work-break time
Up to 50%	0 days	39 days
> 50% up to 100%	1	37
> 100% and ≤ 150%	4	34
> 150% and ≤ 200%	7	32
> 200% and ≤ 250%	13	29
> 250% and ≤ 300%	17	27
> 300% and ≤ 400%	51	16
> 400% and ≤ 500%	55	15
> 500% and ≤ 600%	60	14
> 600%	114	5

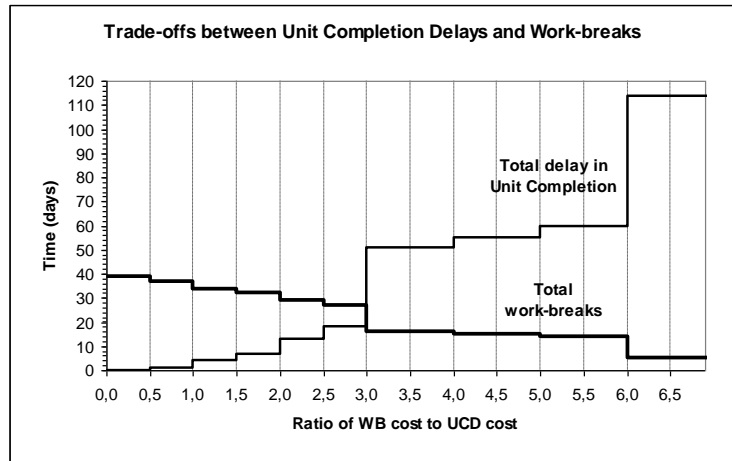** as deviations from minimum completion time*

Figure 4. Trade-offs between unit completion delays and work-breaks.

5. Conclusions

Scheduling of linear repetitive construction projects is not a single dimension decision process. Factors such as the duration of the project the delivery of the individual project units on time, the continuation of resource usage, and the slack time are factors that construction managers must take into consideration in deciding an optimum scheduling for the project. A scheduling decision must take into consideration more than a single factor and most of the times tradeoffs are required between unit completion times, project duration and work-breaks. Strictly scheduling methods like CPM and RSM can provide mainly answers regarding time dimension but cannot address in an integrated way, cost considerations that are raised since many cost factors such different cost of work-breaks at each task, penalty costs related to delays in project duration, financial costs arising from inadequate cash flow because of lateness in delivery of partial units. The MOLP-LRP model can address these issues and provide answers to relative questions. Furthermore the MOLP-LRP can be used to determine the optimum number of segments in a linear project when segments are not defined as physical units (i.e. floors, apartments etc.) given that an increase in the number of units affects positively the duration of the project in a way of diminishing returns but also increases the cost of work-breaks and the total employment of the resources.

5. References

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