Third International Conference on Construction in the 21st Century (CITC-III)
"Advancing Engineering, Management and Technology"
15-17, September 2005, Athens

Analysis of Vibrations of Lightweight Floor Systems

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Abstract
During the last four decades floor systems used in housing and office-buildings in the Netherlands were mostly made of stone-like materials, and can be characterized as heavy. In recent years, in light of sustainable building methods, the trend is to reduce the use of materials and thus build lighter. Lightweight floor structures are however often found to be more susceptible to vibrations than heavier floor structures. The vibrations are caused by dynamic actions such as walking persons or vibrating machines such as a washing machine. This represents the floor system as a beam supported by hinges with a rotational spring at both ends. The influence of the parameters involved are described. An analytical approach is used that results in an approximation formula to find the first natural frequency depending on several parameters and recommendations for practical use are given.

Keywords
Vibrations, Lightweight Floor systems, Industrial Flexible and Demountable building, Rotation spring support

1. Introduction

Traditionally floor systems used in housing and office-buildings in the Netherlands were made of stone-like materials. These floor systems, which can be characterized as heavy, normally posed little problems concerning vibrations. In recent years, in light of sustainable building methods, the trend is to reduce the use of materials and thus build lighter. Lightweight structures are however often found to be susceptible to vibrations. The vibrations are caused by dynamic actions such as walking persons or vibrating machines such as a washing machine. When one of the natural frequencies of the floor system, usually the first, is close to the frequency of excitation, problems can occur. Usually it is found that the higher the first natural frequency, the better the performance.

The vibration behavior of beams for several boundary conditions is well described in literature [Weaver et al, 1990]. The cases discussed are mostly those with free or completely fixed end conditions. Hibbeler [Hibbeler, 1975] discussed the case of a prismatic beam with rotational spring supports and presented a formula that could numerically be solved.

This paper discusses the case of a prismatic beam with rotational spring end supports and presents an approximation formula to find the first natural frequency. This allows for greater possibilities for analysis of the discussed case. Also a parametric study will be presented that will show the influence of relevant parameters on the natural frequency and recommendations for the design of lightweight floor systems will be given.

1.

2. Analytical model

A single span floor system can be regarded as a beam supported at both ends. In the engineering practice it is often assumed the supports are free hinges. However in most cases this is not true because the support is partly fixed as schematized in Figure 1.

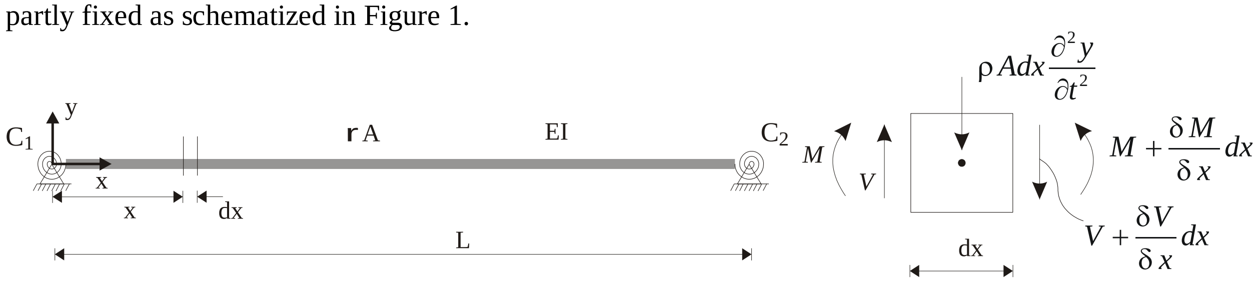


Figure 1, Scheme of structure

Where:

L	[m]	: Length of the beam between the supports
EI	[Nm^2]	: Bending stiffness of the beam
2. ρA	[kg/m’]	: Mass per unit length, acting as a distributed load
C_1, C_2	[Nm/rad]	: Rotational stiffness of left and right support respectively

2.1 Exact solution for the natural frequency

The exact solution for the natural frequency of the beam with rotational spring supports can be found by solving the differential equation (1.1), and application of the boundary conditions.

The differential equation [Weaver, et al 1990] governing this structure is:

$$EI\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} = 0 \tag{1.1}$$

with y = deflection of the beam, t = time
The deflection of a beam in the first natural mode can be written as:

$$y(x,t) = Y(x)(E\cos\omega t + F\sin\omega t) \tag{1.2}$$

with Y(x) decribes the deflection of the beam as a function of x, ω = angular velocity in rad/s and E and F are constants. Introducing the parameter β as a measure for the frequency given by,

$$\beta^4 = \omega^2(\rho A / EI) \qquad [\mathrm{m}^{-1}\ \mathrm{rad}^{1/2}] \tag{1.3}$$

equation (1.1) combined with equation (1.2) and (1.3) we obtain:

$$\frac{d^4Y}{dx^4} - \beta^4 Y = 0 \tag{1.4}$$

Equation (1.4) can be solved for β, by letting Y taking the form

$$Y(x) = A\sin\beta x + B\cos\beta x + C\sinh\beta x + D\cosh\beta x \tag{1.5}$$

The constants A, B, C and D can be determined by using the boundary conditions for the case under consideration. Hibbeler [Hibbeler, 1975] introduced two parameters, u_1 and u_2, to group the parameters that influence the boundary conditions.

$$u_1 = \frac{C_1 L}{EI}\mathrm{rad}^{-1}, \quad u_2 = \frac{C_2 L}{EI}\mathrm{rad}^{-1} \tag{1.6}$$

Using these, the boundary conditions can be written as follows.

At x = 0 deflection: $Y(0) = 0$

moment force: $C_1 Y'(0) = EIY''(0) \quad \Rightarrow \quad \frac{u_1}{L} Y'(0) = Y''(0)$ (1.7)

At x = L deflection: $Y(L) = 0$

moment force: $-C_2 Y'(L) = EIY''(L) \quad \Rightarrow \quad -\frac{u_2}{L} Y'(L) = Y''(L)$ (1.8)

Combining (1.5), (1.7) and (1.8) and substituting R=*βL*, an equation depending on only 3 variables is found with R [$\mathrm{rad}_{1/2}$]as a measure for the natural frequency

$$\begin{aligned} &-u_1 R\sinh(R)\cos(R) + 2R^2\sinh(R)\sin(R) + u_1 R\cosh(R)\sin(R) \\ &- u_1 u_2\cosh(R)\cos(R) + u_2 R\cosh(R)\sin(R) + u_1 u_2 - u_2 R\cos(R)\sinh(R) = 0 \end{aligned} \tag{1.9}$$

Equation (1.9) has only one unknown variable, R, which has to be solved for. However this unknown is not explicit so it cannot be solved analytically. More than one value of R can be found for a combination of parameters, represented by u_1 and u_2, and represent the different modes of vibration. Using numerical methods values for R can be found.

1.2 Approximation function

In this paper the focus is primarily on the first natural frequency. In practice the first natural frequency is the most important for analyzing floor systems, as this will generally be in the range of the excitation frequency that is below 7 Hz. Higher order frequencies are of less importance. Though the exact solution of (1.9) will result in accurate values for the first natural frequency, it is not adequate for practical use as one has to use numerical techniques to find a solution. A solution that results in a formula that gives the first natural frequency explicitly is desired. As stated above this cannot be achieved analytically, but it proves possible to find a very accurate approximation function where R is indeed explicit. The approximation function for R, called $\tilde{R}$ will be taken in the form of

$$\tilde{R}_{(u_1,u_2)} = \frac{S_1(u_1+u_2)+S_2(u_1u_2)+S_5}{S_3(u_1+u_2)+S_4(u_1u_2)+1} \qquad (1.10)$$

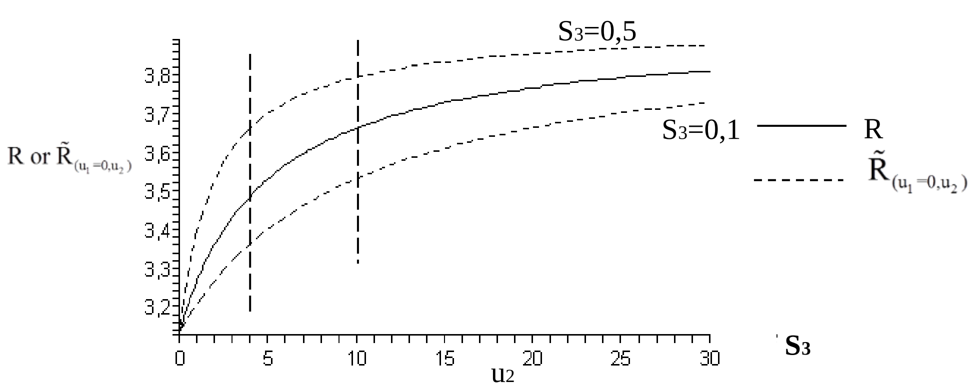
This function results in generally the same type of graph as with the exact formula. This function has an adequate ability to fit the curve of Figure 2 by choosing adequate values for the five constants, S_1 to S_5. These constants can be found by examining the limit cases as will be shown below. In this approximation function five constants have to be determined. As switching the left and the right side of the structure, resulting in swapping u_1 and u_2, should yield the same value for $\tilde{R}$ the constants for u_1 and u_2 of the same order have to be equal and thus are grouped together.

The first constant to be solved is S_5 and can be solved by examining the limit case where $u_1=u_2=0$. This reduces (1.10) to $\tilde{R}_{(u_1=u_2=0)} = S_5$. Numerically solving (1.9) for $u_1=u_2=0$, results in the value $S_5=\pi$, which is obvious as this represents the case of an unrestrained beam. The quotient S_1 / S_3 can be found by examining the limit state where $u_1=0$ and $u_2 \to \infty$, which represent a beam fixed at one end and freely supported at the other end. Solving (1.9) numerically for this case, (1.10) reduces to

$$\tilde{R}_{(u_1=0;u_2\to\infty)} = \frac{S_1}{S_3} = 3.92660 \qquad (1.11)$$

Expressing S_1 as a function of S_3 and assign $u_1=0$ we can rewrite (1.10) as follows:

$$\tilde{R}_{(u_1=0,u_2)} = \frac{3.92660S_3(u_2)+\pi}{S_3(u_2)+1} \qquad (1.12)$$



3.

Equation (1.12) will be correct for the extreme values of $u_2=0$ and $u_2 \to \infty$, combined with $u_2 \to \infty$, regardless of the value chosen for S_3, but has also to be optimal for all values of u_2 in between. This can be done by choosing strategically a value for u_2 where equations (1.9) and (1.12) have to be equal. The graphs of these to equations, using different values for S_3 are shown in Figure 3. The value chosen for u_2 to calculate S_3 is a value where the difference, ΔR, between (1.9) and (1.12) is largest. The range for suitable values of

u_2 is shown in Figure 3 by the dashed vertical lines. The value for u_2 of 7 proves to result in the best approximation. After numerically solving (1.9) for u_2 = 7 we obtain,

$$\tilde{R}_{(u_1=0;u_2=7)} = \frac{3.92660 S_3(7) + \pi}{S_3(7)+1} = 3.59933 \qquad \Rightarrow \qquad S_3 = 0.19981,\ \ S_1 = 0.78457 \tag{1.13}$$

It can be shown that S_2 and S_4 can be found by examining the limit case were $u_1 = u_2 \rightarrow \infty$. The resulting approximation function is:

$$\tilde{R} = \frac{0.78457(u_1+u_2) + 0.15976(u_1 u_2) + \pi}{0.19981(u_1+u_2) + 0.03377(u_1 u_2) + 1} \tag{1.14}$$

2.3 Validation of approximation function

The derived approximation function (1.14) has a deviation compared to the exact solution, given implicitly by equation (1.9). In this section we will show the distribution of this deviation, ΔR, and express it in percent. ΔR at given values of u_1 and u_2 is defined by:

$$\Delta R = \frac{R^2(u_1,u_2) - \tilde{R}^2(u_1,u_2)}{\tilde{R}^2(u_1,u_2)} \cdot 100\% \tag{1.15}$$

We can now use the solution found by (1.9) for R and $\tilde{R}$ given by the approximation function (1.14) with equation (1.15). This results in an equation depending on only three variables, being ΔR, u_1 and u_2. This equation is plotted in Figure 3. It can be seen from this figure that the maximum error is about 0.07% and this occurs only for small values of u_1 and u_2 between 0 and 10. For larger values the error reduces to 0%, which of course should be the case as we determined the constants S_1 through S_5 by using the limit cases. It can also be seen from the error distribution that the values chosen to determine constants S_1 and S_3 were correct as the maximum positive error equals the maximum negative one.

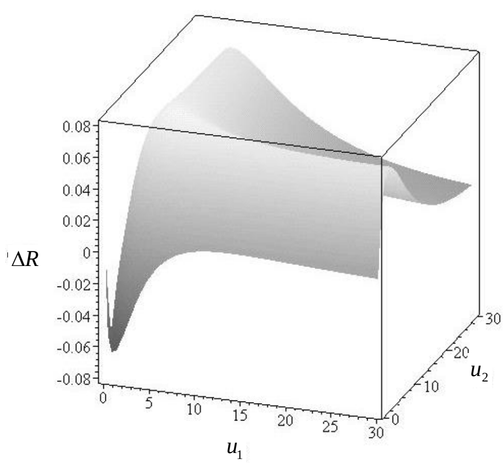


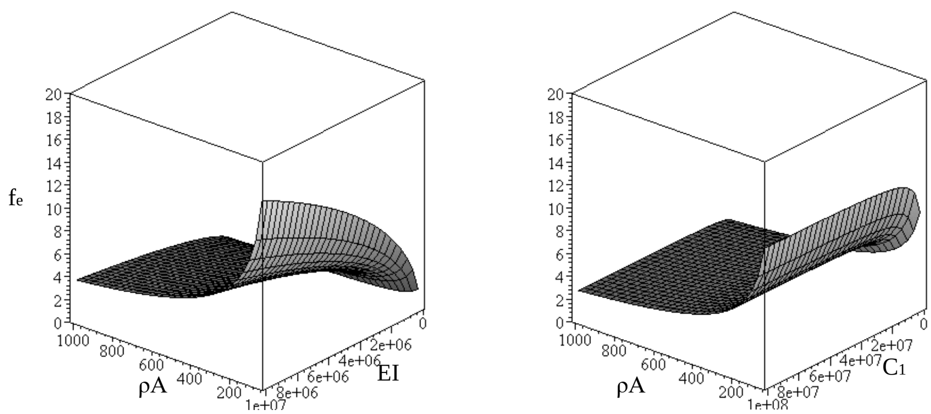
Figure 3, Distribution of deviation according to (1.15) for approximation formula (1.14)

3 Results

The frequency can be found combining equations (1.3), (1.14) and $f_e = \omega/(2\pi)$:

$$f_e = \frac{\tilde{R}^2_{(L,EI,C_1,C_2)}}{2\pi L^2}\sqrt{\frac{EI}{\rho A}} \quad \text{[Hz]} \qquad (1.16)$$

A range of values of practical relevance for the parameters involved has been determined. Graphs can be made showing the influence of two parameters on the natural frequency while choosing a constant value for the other parameters. Figure 4 shows two graphs, showing the influence of the mass of the beam (ρA) combined with the stiffness (EI) and the rotation spring stiffness on one end (C_1) respectively. Other graphs can be drawn showing the influence of the parameters on the frequency but also on each other.



4 Conclusions

- An accurate approximation function for the first natural frequency of a beam with rotational spring supports has been derived
- Lightweight floors systems benefit more from rotational spring stiffness at the supports than heavier floors.
- The majority of the possible increase of the natural frequency, due to rotational springs, can be obtained by relatively small rotational spring stiffness.
- Lightweight floors benefit more from higher stiffness than heavier floors.

5. References

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