

# **Optimal Construction Design using the Harmony Search Algorithm as an Optimization Tool**

Nicolaos Theodossiou, Diamantis Karakatsanis  
*Aristotle University of Thessaloniki, Greece*

Ioannis Kougias  
*EC– Joint Research Centre, Italy*

## **Abstract**

The determination of the optimal values of the numerous parameters involved in the design of complex construction elements has always been a challenge for engineers. The use of optimization approaches and techniques has gradually replaced the practice of empirical determination of crucial design parameters. The quest for more robust but at the same time user-friendly optimization techniques is nowadays more intense than ever. This need to develop and apply new optimization techniques in order to achieve more efficient and effective solutions in complex management and design problems, has led to the introduction of a series of interesting new methods. The meta-heuristic method of music-based Harmony Search Algorithm (HSA) is one of the leading optimization techniques nowadays with applications covering a wide range of scientific fields. In this paper, the main features of this method are presented along with a specific construction design application. This application is performed using specially-designed computer software programmed in MATLAB environment. The created software includes a user-friendly GUI (Graphical User Interface) that allows the user to easily interact with the optimization tool.

## **Keywords**

optimization, metaheuristics, decision making, structural design, harmony search algorithm

## **1. Introduction**

Structural design applications involve choosing an option among many alternatives, which is equal to determining the solution with the more positive and less negative technical characteristics. The procedure to determine this solution is called optimization. Obviously, the simplest optimization technique is an exhaustive “trial and error” series, where the best solution is determined after trying and evaluating the all possible alternatives.

In this paper, the authors present the potential of metaheuristics in structural design optimization. In order to achieve this goal the authors designed and developed a Harmony Search Algorithm in Matlab environment and applied it to a typical truss design application. The results have been compared to those obtained by an exhaustive, time-consuming algorithm that explores every possible solution of the search space. In the last section, a discussion on the findings underlines the advantages of the application of metaheuristics in similar applications.

Structural design optimization is of critical importance to the constructions because a careful design may produce safer structures, saving time and financial resources at the same time, through the efficient use of optimization techniques. Although traditional mathematical methods such as linear and dynamic programming have been used and developed long time ago, it was only in the late 20<sup>th</sup> century when stochastic metaheuristic algorithms launched a new era in the field. These methods introduced sophisticated randomness in order to spot optimal solutions. Their advantages are that they can solve demanding optimization problems in short time and with relatively low programming complexity. As a

result, after their initial appearance, their development and implementations in almost every scientific field has been extensive.

### **1.1 Metaheuristic optimization algorithms**

Metaheuristic algorithms as a rule are inspired by natural or artificial procedures. In that sense they imitate natural or artificial phenomena that continuously advance to better states in order to carry out internal search processes.

Genetic Algorithms (Holland, 1973) is probably the most wide-spread optimization technique and imitates the natural evolution process according to Darwin's theory. Many methods known as evolutionary computational methods, such as Evolution Strategies, Evolutionary Programming, Genetic Programming, are based on the principle of evolution. Simulated Annealing is a successful algorithm developed in the early 80's relating to an artificial phenomenon, the metals' characteristic of recrystallizing in an annealing process (Kirckpatrick, 1983). The interest of those involved in optimization continued to be sustained in developing new algorithms such as the Particle Swarm Optimization (Eberhart and Kennedy, 1995) and Ant Colonies (Dorigo, 1996), which were inspired by the behavior of living organisms.

Geem in 2001 introduced Harmony Search Algorithm (HSA), a modern metaheuristic algorithm inspired from the music creation process (Geem et al., 2001). HS Algorithm, which is used in the present study, is a powerful and efficient tool with the extra advantage of having a simple structure. These characteristics attracted the interest of those involved in the optimization field. Initially HSA was designed for the optimum design of water distribution networks (Geem et al., 2002). Since then, there has been sustained and increasing interest in HSA applications. In the current literature, apart from water engineering optimization problems, one can find a vast variety of interesting implementations (Kougias and Theodossiou, 2010).

### **1.2 State-of-the-Art**

In the current literature one can find several applications of the Harmony Search Algorithm on structural design optimization. Lee and Geem (2004) solved eight different design problems, using HSA. All eight problems dealt with the optimal design of steel truss structures of varying complexity. Thus, the algorithm was applied to trusses comprising from 10 to 200 steel bars. In 2005, Lee et al. solved additional complex steel truss structure problems using HSA. Their application was preceded by a calibration-validation of the HS algorithm using the Rosenbrock test-function. The obtained results were compared to the bibliography, illustrating the efficiency of HSA to solve such problems. Mahdavi et al. (2007) developed a variant of the HSA and used it for both structural and mechanical parts' design. Fesanghary et al. (2008) hybridized HSA with sequential quadratic programming in order to optimize structural design problems. Similar was also the approach of Kaveh and Talatahari (2009) who hybridized HSA with Particle Swarm and Ant Colonies and solved the same problems. Saka (2009) presented the optimum design of steel frames with the use of HSA. Degeterkin (2012) developed two variants of HSA, the 'Efficient HSA' and the 'Self-Adaptive HSA' and solved four classical truss structure weight minimization problems in order to demonstrate the robustness of the proposed algorithms. Gholizadeh and Barzegar (2013) presented an HSA algorithm that simultaneously deals with size and shape optimization of structures with frequency constraints. Maheri and Marimani (2014) presented a variant of HSA that proposes a different handling of solutions storage to memory. They applied it to steel-frame design problems.

## **2. Harmony Search Algorithm**

### **2.1 Relationship between music and mathematics**

The relationship between music and mathematics has been close since the ancient times. Mathematicians tried to interpret the governing rules of mathematics using the art of music. On the other hand, composers tried to use mathematics in order to deeply understand music.

During recent times, since the Baroque period, this bond has been strengthened. Sometimes as a conscious effort by musicians-composers and sometimes as part of a rumored and almost mystical relationship, mathematics and music came closer. Iannis Xenakis represents a special example. His deep knowledge both in mathematics and music is illustrated in his work on the use of mathematical functions to compose music (1992) distinguishing him among the most eminent music figures of the 20<sup>th</sup> century.

## **2.2 Analysis of Harmony Search Algorithm (HSA)**

### **2.2.1 The basic elements of the algorithm**

The Harmony Search Algorithm is a stochastic meta-heuristic method based on the sequential production of possible solutions. It belongs to the category of “neighborhood meta-heuristics” that produce one possible solution (called “harmony”) in each iteration. Every possible solution consists of a set of values of the decision variables of the function that needs to be optimized. During the optimization process, a number of “harmonies” equal to the “Harmony Memory Size” are stored in the “Harmony Memory” (HM), a database that includes the produced set of solutions. The optimization process is completed as soon as the predefined total number of iterations has been achieved (Geem, 2001).

### **2.2.2 Characteristics of the Harmony Search Algorithm**

Following the definition of the decision variables, the Harmony Memory matrix is formulated. Harmony Memory is  $m \times n$  matrix, where  $m$  is the Harmony Memory Size and  $n$ , the number of decision variables included in the objective function. Then, the algorithm begins producing and evaluating new “Harmonies” through the application of HSA’s basic mechanisms:

1. Harmony Memory Consideration uses variables’ values already stored in the Harmony Memory. This mechanism ensures that good solutions located during the optimization process will contribute to the formation of even better solutions.
2. Some of the solutions selected by the Harmony Memory Consideration mechanism will be slightly altered. This is the second mechanism of the algorithm named Pitch Adjustment and it is performed by selecting a neighboring values of the decision variables
3. The third mechanism is Improvisation, which introduces new, random elements to the solutions. The probability of introducing such random values is  $(100 - \text{HMCR})\%$ . In this way the variability of solutions is enriched.

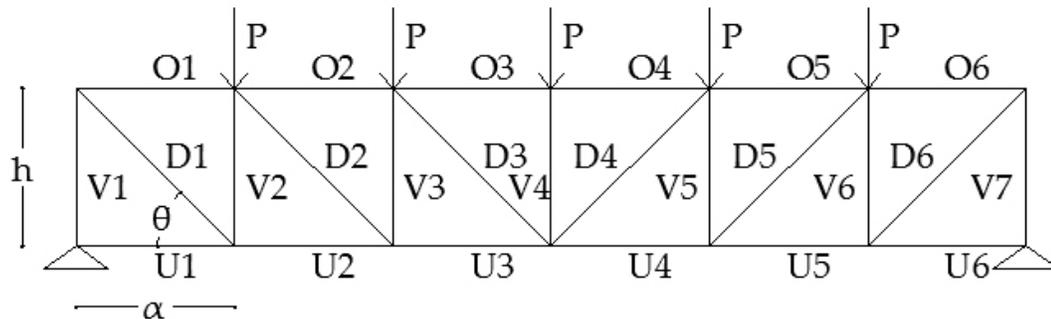
After the creation of a new “Harmony”, its performance is evaluated according to the corresponding value of the objective function. If this performance is better than that of the worst “Harmony” stored in the Harmony Memory, it replaces it. This procedure is repeated until the ending criterion, is reached.

## **3. Case study**

Decision-making in structural design applications often involves the solution of NP-hard problems with several and –as a rule- conflicting objectives. Solving such problems using exhaustive algorithms requires long runs with a duration that increases exponentially with the problem’s complexity. Thus, using a non-deterministic method although it doesn’t ensure the detection of optimal solutions (members of the Pareto front in multiobjective optimization), it provides an efficient, practical method to solve complex problems. Heuristics and Metaheuristics such as the HSA can be used to solve NP-hard problems in polynomial time.

### 3.1 Application in a steel-truss

In the present section a 25 bar planar steel-truss design is optimized. The structure is symmetric and isostatic and doesn't allow any movement outside the x-y plane. The truss components have different intersection areas, according to their position. Thus the upper bars  $O_i$  have a uniform intersection of  $X_1$  area, the lower bars  $U_i$ , an intersection  $X_2$ , the vertical bars  $V_i$ , a uniform area  $X_3$  and the diagonal bars  $D_i$ , an intersection area  $X_4$ . The geometry of the truss, as well as the loads applied, are illustrated in Figure 1.



**Figure 1: The steel-truss model**

The calculation of the stress in each bar of the truss-structure is an easy task using a relative software or basic structural analysis. The stress of each bar is shown in Table 1 as a function of the load  $P$ , the height of the structure  $h$  and the distance between the vertical bars  $a$ .

$U_{i,p}$	$O_{i,p}$	$V_{i,p}$	$D_{i,p}$
$U_1 = 0$	$O_1 = -2.5Pa/h$	$V_1 = -2.5P$	$D_1 = 2.5P/\sin\theta$
$U_2 = 2.5Pa/h$	$O_2 = -4Pa/h$	$V_2 = -2.5P$	$D_2 = 1.5P/\sin\theta$
$U_3 = 4Pa/h$	$O_3 = -4.5Pa/h$	$V_3 = -1.5P$	$D_3 = 0.5P/\sin\theta$
$U_4 = 4Pa/h$	$O_4 = -4.5Pa/h$	$V_4 = -P$	$D_4 = 0.5P/\sin\theta$
$U_5 = 2.5Pa/h$	$O_5 = -4Pa/h$	$V_5 = -2.5P$	$D_5 = 1.5P/\sin\theta$
$U_6 = 0$	$O_6 = -2.5Pa/h$	$V_6 = -1.5P$	$D_6 = 2.5P/\sin\theta$
		$V_7 = -2.5P$	

**Table 1: Stress of each bar of the truss-structure (Load P)**

The innovation of the present application compared to those in the bibliography (section 1.2) is that it deals with a multiobjective application with two conflicting objective functions. The first objective is the minimization of the structures weight, while the second includes minimizing the vertical settlement of its middle truss. Since in such problems a unique optimal solution doesn't exist, the efforts will focus on identifying a set of non-dominated solutions, known as the Pareto front.

### 3.2 Objective functions

The total structure's weight is calculated using the following equation:

$$F_1 = d \cdot (6 \cdot a \cdot X_1 + 6 \cdot a \cdot X_2 + 7 \cdot h \cdot X_3 + \frac{7 \cdot h \cdot X_4}{\sin\theta}) \quad (1)$$

where  $d$ , is the density of the bars' material

In order to calculate the vertical displacement, the classic structural analysis method has been used with a unit (dummy) load moving on the top bars. The results of this analysis are presented in Table 2.

$U_{i,1}$	$O_{i,1}$	$V_{i,1}$	$D_{i,1}$
$U_1=0$	$O_1=-0.5/\tan\theta$	$V_1=-0.5$	$D_1=0.5/\sin\theta$
$U_2=0.5/\tan\theta$	$O_2=-1/\tan\theta$	$V_2=-0.5$	$D_2=0.5/\sin\theta$
$U_3=1/\tan\theta$	$O_3=-1.5/\tan\theta$	$V_3=-0.5$	$D_3=0.5/\sin\theta$
$U_4=1/\tan\theta$	$O_4=-1.5/\tan\theta$	$V_4=0$	$D_4=0.5/\sin\theta$
$U_5=0.5/\tan\theta$	$O_5=-1/\tan\theta$	$V_5=-0.5$	$D_5=0.5/\sin\theta$
$U_6=0$	$O_6=-0.5/\tan\theta$	$V_6=-0.5$	$D_6=0.5/\sin\theta$
		$V_7=-0.5$	

**Table 2: Stress of each bar of the truss-structure (unit “dummy” Load)**

The vertical displacement at the center of the truss is calculated by the following equation 2:

$$F2 = \sum \left( \frac{U_{i,p} \cdot U_{i,1} \cdot a}{E \cdot X_2} + \frac{O_{i,p} \cdot O_{i,1} \cdot a}{E \cdot X_1} + \frac{V_{i,p} \cdot V_{i,1} \cdot h}{E \cdot X_3} + \frac{D_{i,p} \cdot D_{i,1} \cdot h / \sin\theta}{E \cdot X_4} \right) \quad (2)$$

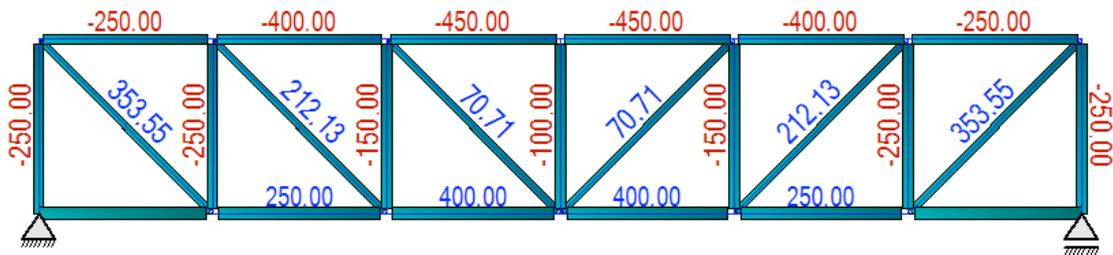
### 3.3 Constraints

F1 and F2 equations are the objective functions of the problem. Regarding the problem's constraints those bars that are under compressive stress ( $O_i$  and  $V_i$ ) shouldn't receive a stress higher than the buckling limit. The critical buckling stress is calculated using Equation 3 ( $L_i$  is the length of each bar):

$$P_{cr} = \frac{\pi^2 EI}{L_i^2} \quad (3)$$

### 3.4 Structural analysis

The upper limit for the stress of bars under tension ( $U_i$  and  $D_i$ ) is the yield stress  $\sigma_y$ . This constraint also applies to bars under compressive stress. Solving the truss bar values:  $P=100$  kN,  $a=2$  m,  $h=2$  m,  $\theta=45^\circ$  results to bar stresses illustrated in Figure 2.



**Figure 2: Stress on the bars of the studied structure**

## **5. Application of the optimization model**

### **5.1 Single-objective and multiobjective optimization**

The original Harmony Search Algorithm (Geem, 2001) aimed to solve optimization problems of one objective. Extending HSA's applicability to multiobjective problems is not a simple task, because it involves re-designing its elements. Multiobjective problems have as a rule complicated structure, conflicting objectives and the simultaneous optimization of the different functions can be a challenging task. Besides, the global optimum of the multiobjective problem is usually very different from the solutions that optimize each objective.

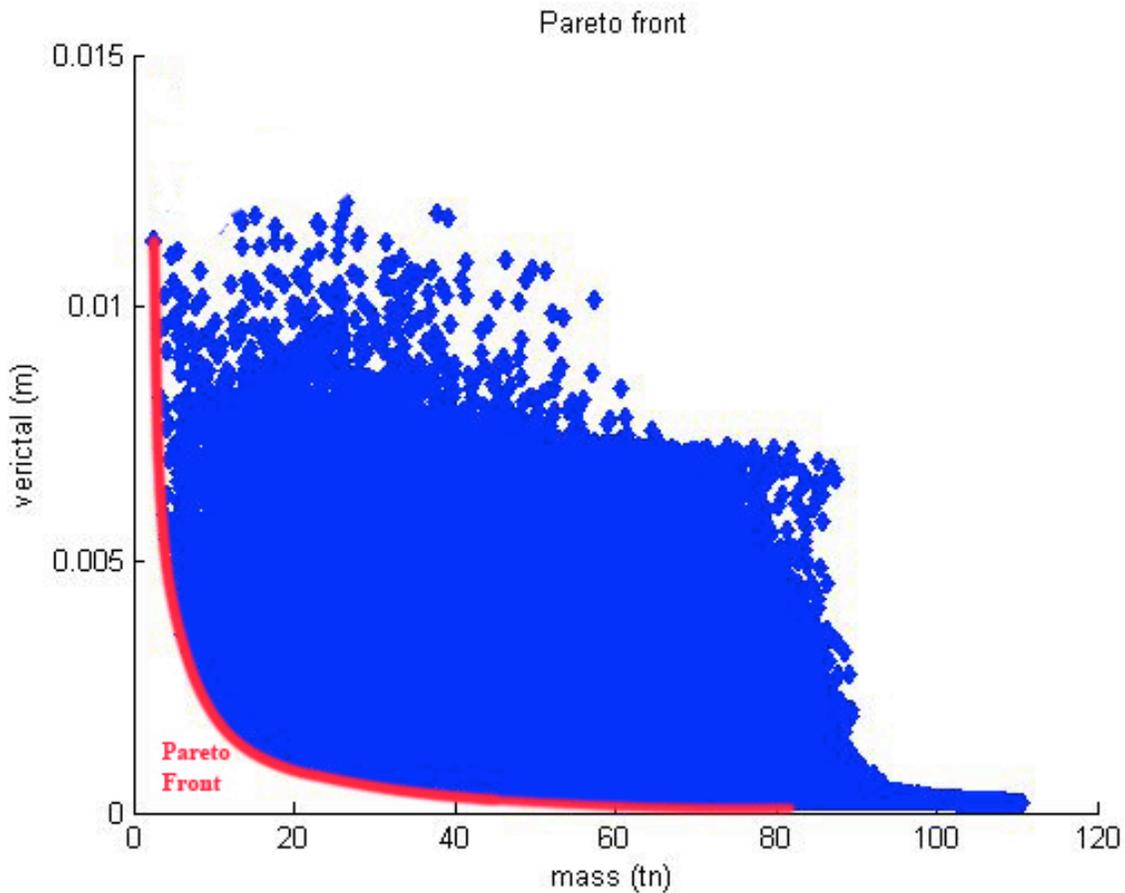
As already mentioned, multiobjective problems can be analyzed using the Pareto domination theory. According to that theory a solution is dominating another solution if only it is better according to at least one objective function and (at least) equal according to the other objectives.

The authors have developed different multiobjective variants of the HSA analyzed in Kougiyas et al. (2012) and in Kougiyas & Theodossiou (2013). In order to keep in balance all objectives all the developed algorithms include sub-programs that guarantee equal importance of all objectives during the selection processes. Such mechanisms also support the algorithm's effort to find solutions that cover all the width of the Pareto front.

### **5.2. Problem's optimum structural design**

In figure 3 the set of non-dominated solutions for the studied problem is illustrated. The complete set of solutions forms a curve (shown in figure 3 with a red line) known as the Pareto front. The decision maker can choose different solutions from those along the Pareto front since all these solutions are equally optimal. Obviously any reduction in the mass will result to compromises to the vertical settlement of the structure and an increase in the displacement. Accordingly structures with smaller vertical settlement will need to be heavier.

It is important, however, that obtaining the Pareto front through the implementation of the presented methodology and algorithm, provides the decision maker with a great number of options for structural design. Thus, the decision maker can select the design that is more suitable to the particularities of the structure and location from a wide range of options.



**Figure 3. The Pareto front (red line) of the optimal solution**

### 5.3. Conclusions

In the present paper a classic structural design problem has been designed. A methodology to optimize the design variables of the structure has been analytically presented. This methodology uses Harmony Search Algorithm, a metaheuristic optimization technique in its core.

An important contribution is the expansion of the HSA to solve multi-objective problems that have several targets and better simulate real-world applications.

### 6. References

- Degertekin, S. O. (2012). Improved harmony search algorithms for sizing optimization of truss structures. *Computers & Structures*, 92, 229-241.
- Dorigo, M., Maniezzo, V., & Colomi, A. (1996). Ant system: optimization by a colony of cooperating agents. *Systems, Man, and Cybernetics, Part B: Cybernetics*, IEEE Transactions on, 26(1), 29-41.
- Eberhart, R. C., & Kennedy, J. (1995). A new optimizer using particle swarm theory. In *Proceedings of the sixth international symposium on micro machine and human science (Vol. 1, pp. 39-43)*.
- Fesanghary, M., Mahdavi, M., Minary-Jolandan, M., & Alizadeh, Y. (2008). Hybridizing harmony search algorithm with sequential quadratic programming for engineering optimization problems. *Computer methods in applied mechanics and engineering*, 197(33), 3080-3091.

- Geem, Z. W., Kim, J. H., & Loganathan, G. V. (2001). A new heuristic optimization algorithm: harmony search. *Simulation*, 76(2), 60-68.
- Geem, Z. W., Kim, J. H., & Loganathan, G. V. (2002). Harmony search optimization: application to pipe network design. *International journal of modelling & simulation*, 22(2), 125-133.
- Holland, J. H. (1973). Genetic algorithms and the optimal allocation of trials. *SIAM Journal on Computing*, 2(2), 88-105.
- Gholizadeh, S., & Barzegar, A. (2013). Shape optimization of structures for frequency constraints by sequential harmony search algorithm. *Engineering Optimization*, 45(6), 627-646.
- Kaveh, A., & Talatahari, S. (2009). Particle swarm optimizer, ant colony strategy and harmony search scheme hybridized for optimization of truss structures. *Computers & Structures*, 87(5), 267-283.
- Kirkpatrick, S., Gelatt, C. D., & Vecchi, M. P. (1983). Optimization by Simulated Annealing. *Science*, 220(4598), 671-680.
- Kougias, I., and Theodosiou, N. (2010). A new music-inspired harmony based optimization algorithm: Theory and applications. In Proceedings of: International Conference on Protection and Restoration of the Environment X, Corfu.
- Kougias, I., Katsifarakis, L., & Theodossiou, N. (2012). Medley Multiobjective Harmony Search Algorithm: Application on a water resources management problem. *European Water*, 39, 71-52.
- Kougias, I., & Theodossiou, N. (2013). Multiobjective pump scheduling optimization using harmony search algorithm (HSA) and polyphonic HSA. *Water Resources Management*, 27(5), 1249-1261.
- Lee, K. S., & Geem, Z. W. (2004). A new structural optimization method based on the harmony search algorithm. *Computers & Structures*, 82(9), 781-798.
- Lee, K. S., Geem, Z. W., Lee, S. H., & Bae, K. W. (2005). The harmony search heuristic algorithm for discrete structural optimization. *Engineering Optimization*, 37(7), 663-684.
- Mahdavi, M., Fesanghary, M., & Damangir, E. (2007). An improved harmony search algorithm for solving optimization problems. *Applied Mathematics and Computation*, 188(2), 1567-1579.
- Maheri, M. R. & Narimani, M.M (2014). An enhanced harmony search algorithm for optimum design of side sway steel frames. *Computers & Structures*, 136 (2014): 78-89.
- Saka, M. P. (2009). Optimum design of steel sway frames to BS5950 using harmony search algorithm. *Journal of Constructional Steel Research*, 65(1), 36-43.
- Xenakis, I. (1992). Formalized music: thought and mathematics in composition (No. 6). Pendragon Press.