

# 1 Modelling correlations in highway construction projects

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5 **Abstract.** The study examines the different types of correlations between the  
6 construction costs, times and cost/time of highway construction projects and  
7 quantified their impacts on the total cost and total time of different structures of  
8 highway construction projects to determine whether repetitions of activities  
9 amplify the impact of correlation on construction cost and time. Highway  
10 construction projects are often plagued by cost and time underestimation due to  
11 ignorance of costs and times correlation between the activities in highway  
12 projects when deterministic estimation techniques are employed. Therefore, a  
13 probabilistic model is adopted in estimating the effect of correlations on the  
14 probability distributions of total cost and total time. This study identified various  
15 types of correlations between the costs, times and cost/time in the construction  
16 of highway projects and modelled them using Gaussian copula and analyzed the  
17 impact of such correlations in the construction of a highway project using Monte  
18 Carlo simulation. The results show that the standard deviation of the total cost  
19 and total time increases with the magnitude of the correlation and type of  
20 correlation matrix and, most importantly, it increases considerably with the  
21 number of costs and times that are correlated. Based on these findings, the study  
22 concludes that the deterministic estimation technique in use does not capture the  
23 wide range of the possible total cost and time of highway projects resulting in  
24 significant overruns.

25 **Keywords:** correlation, cost and time underestimation, cost and time overruns,  
26 deterministic estimation, highway construction, repeated activity

## 27 1 Introduction

28 Transportation infrastructure projects, particularly highway construction projects, are  
29 often plagued by cost and time underestimations [1]. Various studies [2-5] proved that  
30 both construction costs and times are positively correlated. Positive correlations caused  
31 the standard deviation of the correlated costs and times to be larger than the standard  
32 deviation of uncorrelated costs and times, and the total construction cost and time of  
33 projects were underestimated if the costs, times and cost/time correlation were  
34 disregarded [2].

35 Correlation measures and describes the strength and direction of the relationship  
36 between two variables [6]. The correlation varies between -1 (fully negatively  
37 correlated) and +1 (fully positively correlated), and, for a correlation equal to 0, the two  
38 variables are uncorrelated. If the value of one variable is above average, the value of

39 the second variable tends to be above average when they are positively correlated, while  
40 it tends to be below average when they are negatively correlated [7].

41 A positive correlation causes the standard deviation of the sum of the correlated  
42 variables to increase since cost and time correlations in construction are usually positive  
43 [2, 4]. Compared to deterministic estimated total cost and total time, the correlated cost  
44 and time are expected to increase the range of possible total costs and total times of the  
45 project, which generates the cloud of points in the total cost and total time directions.

46 Differently from the deterministic cost and time estimate, which corresponds to one  
47 point, modelling the correlation of costs and times provides the range of the possible  
48 total cost and total time by a cloud of points. The cloud of points visually represents the  
49 range of possible correlated total costs and total times of the different structures of the  
50 project.

51 Correlation between the costs and times has been investigated in building  
52 construction [3] and rail line construction [4], but there is a lack of studies on correlation  
53 in highway construction projects. Therefore, this study examines various types of  
54 correlation that occur in a highway construction project and analysed their impact on  
55 the distributions of the cost and time of three main structures of the highway  
56 construction project, namely: earthwork, bridge, and pavement, to determine whether  
57 repetitions of activities amplify the impact of correlations on such distributions.

## 58 **2 Methodology**

59 Correlations in the construction of a highway are analyzed based on the following  
60 methodology steps. First, various types of correlation in highway construction are  
61 identified. After that, the correlation coefficient of each type of correlation is  
62 determined from the analysis of historical data by a panel of experts (which, in the  
63 study, comprised five highway experts with experience of 30 years and more  
64 individually in the construction of highway projects in South Africa recommended by  
65 the South African national roads agency). Identified types of correlation are modeled  
66 and simulated in three structures of highway construction. Finally, the simulation  
67 results are analysed by comparing mean and standard deviation to the independent  
68 results; and findings and conclusions are discussed.

## 69 **3 Correlations in the construction of highway projects**

70 Correlations in the construction of highway projects are investigated on the basis of a  
71 construction network model of a highway. In the network model, the construction of a  
72 highway project is modelled as the construction of a sequence of three main structures,  
73 namely, earthwork, bridge and pavement, each of which is modeled as a sequence of  
74 various construction activities characterized by cost and time.

75 The study identified the following five potential types of correlation in the  
76 construction of highway projects:

77 **Correlation between the costs and times of a repeated activity in a structure**  
78 **(Type 1):** The costs and times of a repeated activity are expected to be positively

79 correlated because of the repetitiveness of the processes in a structure [2]. To analyse  
 80 the impact of repeated activities on the cost and time of activities of each highway  
 81 structure, the intermediate correlation matrix including all positive correlations,  $\rho = (0,$   
 82  $1)$ , is considered, which corresponds to the case where the cost and time are randomly  
 83 selected for each unit, and if one cost or time per unit is above average, the next cost or  
 84 time per unit will also tend to be above average.

$$85 \quad \rho = \begin{bmatrix} 1 & 0.99 & \dots & 0 \\ 0.99 & 1 & 0.99 & \vdots \\ \vdots & 0.99 & 1 & 0.99 \\ 0 & \dots & 0.99 & 1 \end{bmatrix}$$

86 **Correlation between the costs and times of the various activities in a structure**  
 87 **(Type 2):** To evaluate the correlation between the various activities in each of the three  
 88 structures of the highway project, the expert panel of the study considered that the costs  
 89 and times of the various activities in a structure were positively correlated because these  
 90 activities were subjected to the same types of constraints. After analysis of the historical  
 91 data of highway construction projects, the panel of experts recommended the  
 92 correlation coefficients of  $\rho = 0.75$ ,  $\rho = 0.65$  and  $\rho = 0.8$  for earthwork, bridge and  
 93 pavement structures, respectively.

94 **Correlation between the costs and times of activities in adjacent structures**  
 95 **(Type 3):** Although the pairs structures of earthwork-pavement and bridge-pavement  
 96 are adjacent structures, they do not share any activities. Therefore, the expert panel  
 97 recommended no correlation between the costs and times of these pairs. Earthwork and  
 98 the bridge are excavated in the same geology so that a correlation between the costs and  
 99 times of the cutting in earthwork and the cost and time of the bridge excavation is  
 100 expected. However, due to the different excavation methods between the cutting and  
 101 bridge excavation, the panel of experts recommended that the cost and time of  
 102 earthwork and bridge should be independent.

103 **Correlation between the costs and times of same activities in the same types of**  
 104 **structures (Type 4):** Positive correlations between the costs and times of the same  
 105 types of structures (between bridge i and bridge j, between earthwork i and earthwork  
 106 j, and between pavement i and pavement j) were expected if the geology (earthwork,  
 107 bridge) and geometry (bridge) were similar. The cost and time distribution of bridge  
 108 and earthwork structures were a function of the geology and the cost and time of a  
 109 bridge were a function of the geometry so that similar geologies and geometry  
 110 determined similar cost and time distributions.

111 **Correlation between the cost and time of an activity (Type 5):** Due to the  
 112 difficulty of measuring the association between the cost and time of an activity, all  
 113 members of the expert panel agreed to use  $\rho = +0.8$  as correlation coefficient to model  
 114 the correlation between the cost and time of activities.

$$115 \quad \rho = \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}$$

116 Therefore, the study focused on modelling correlation Type 1 (correlation between  
 117 repeated activities), correlation Type 2 (correlation between different activities), and  
 118 correlation Type 5 (correlation between cost and time of the activity), which was  
 119 expected to have a significant effect on the standard deviation and the total cost and  
 120 time of construction of highway projects.

## 121 4 Modelling correlations

122 In this study, the correlation was measured with the non-parametric Spearman  
123 correlation coefficient and modelled with the Gaussian copula. A copula is a  
124 multivariate distribution function defined on the unit cube  $[0, 1]^d$ , with uniformly  
125 distributed marginals [8].

126 The choice of the Spearman correlation from amongst the available correlation  
127 measures and the Gaussian copula from amongst the available correlation models was  
128 because, differently from a random number generator, a copula first generates random  
129 numbers from a uniform distribution, then, through two transformations, it obtains  
130 random numbers correlated with the desired correlations and distributed with the  
131 desired probability distribution. Also, a copula generates the unit costs and production  
132 rates of the construction activities, which are correlated, then these are summed to  
133 obtain the total cost and total time of the activities. In other words, copulas allow the  
134 generation of correlated costs and times with the desired Spearman correlation matrix  
135 and with the desired marginal distribution, and, because of the availability of both  
136 marginal distributions and the correlation matrix for this study, the copula was  
137 employed.

138 The Gaussian copula generation and the summing of the unit costs and production  
139 rates of all activities were repeated for each simulation run. With the copula generation,  
140 the uncertainty in each unit of construction activities was modelled, as well as the  
141 correlation between the costs and times of the activities. At the end of each simulation  
142 run, the costs and the times (critical activities), were respectively summed up to  
143 calculate the correlated total cost and total time of activities for the particular simulation  
144 run. The correlation process was repeated for every simulation run by Monte Carlo until  
145 the standard deviation of total cost and total time of activity bound within  $\pm 1\%$  in 10  
146 out of 10 simulations.

147 A correlated probabilistic analysis was required to model the correlations on the  
148 construction cost and time of activities and evaluate the impact of the identified three  
149 correlations on the total cost and total time of the project. Essentially, the probabilistic  
150 distributions data of the cost and time variables were not available. Therefore, to model  
151 the correlation with copula, the following marginal distributions assumptions were  
152 constructed by the panel of experts based on the results of the variability of cost and  
153 time estimation sessions to generate a correlation probability distribution of costs and  
154 times:

- 155 • The underlying distributions of cost variables and time variables are lognormal  
156 and triangular, respectively [3].
- 157 • The modes of the cost and time distribution are assumed equal to the  
158 deterministic cost and time.
- 159 • The minimum value of the cost and time distributions are 80% of the mode of  
160 each variable's distribution.
- 161 • There is a probability of 2% of exceeding the High Value (which is assumed  
162 to be 150% of the mode) of the cost distributions.
- 163 • The maximum of the time distributions is assumed to be 130% of the mode of  
164 the time distributions.

## 165 5 Results of the study

166 The deterministic estimated cost and time, and resulting means, standard deviations and  
 167 variation of standard deviations of each type of correlation to the uncorrelated case  
 168 (base case) from modelling the three types of correlation on total cost and total time of  
 169 the earthwork, bridge and pavement structures of the selected highway construction  
 170 project are summarized in Table 1.

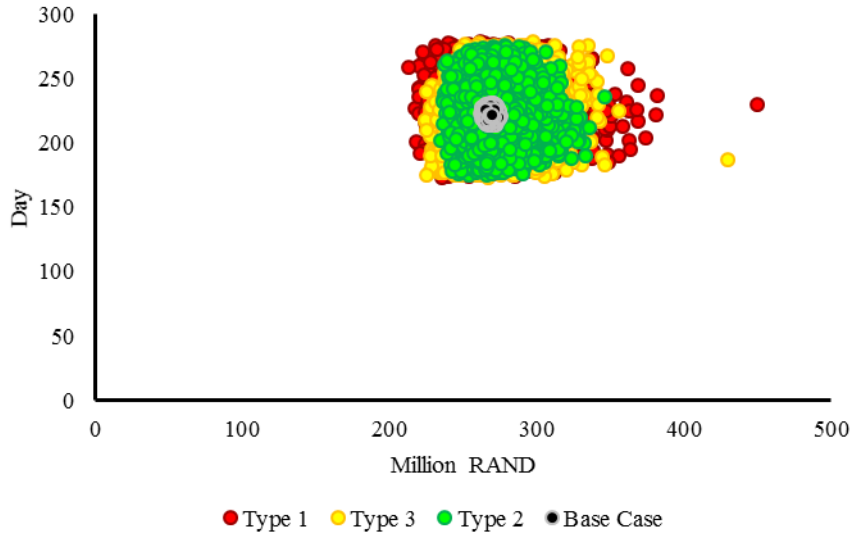
171 The selected case study was a 16.5-kilometre section from a new highway  
 172 infrastructure project with six lanes and controlled-access, which was constructed on  
 173 both flat and hilly terrain in South Africa between 2014 and 2016. The deterministic  
 174 estimated total construction cost of this section was ZAR 809,504,922.80 with a total  
 175 duration of 340 days. The construction of the project comprised 2,973,692 cubic metres  
 176 of earthworks in four different geological conditions, namely, fine, soft, hard and rock,  
 177 46 precast concrete bridges and 1,015,340 cubic metres of asphalt pavements.

178 **Table 1.** Correlated means, standard deviations and standard deviation variations of  
 179 three structures of the highway project

	Correlation	Cost (ZAR)				Time (Days)			
		Deterministic	Mean	Standard Deviation	$\Delta$ Sd	Deterministic	Mean	Standard Deviation	$\Delta$ Sd
Earthwork Structure	Base Case	244,292,292.83	268726606.90	1871424.00	0.00%	215	222.19	2.61	0.00%
	Type 1		268180187.90	22538485.66	1104.35%		222.678	22.34	756.05%
	Type 2		268608078.40	14106038.00	653.76%		222.15	22.22	751.45%
	Type 3		268784679.2	18319948.86	878.93%		222.32	19.53	648.51%
Bridge Structure	Base Case	70,952,700.00	78,030,393.34	1,920,505.95	0.00%	340	351.35	5.04	0.00%
	Type 1		78,113,067.80	6,014,085.33	213.15%		351.36	30.15	498.24%
	Type 2		77,890,401.00	6,425,163.00	234.56%		351.46	32.81	551.42%
	Type 3		78,072,569.32	6,191,307.31	222.38%		351.57	22.61	348.55%
Pavement Structure	Base Case	494,259,930.00	543,664,957.99	4,174,001.38	0.00%	217	224.22	2.64	0.00%
	Type 1		543,853,187.64	52,912,363.32	1167.67%		224.27	13.23	401.14%
	Type 2		543,899,815.54	33,572,597.99	704.33%		224.41	11.13	321.59%
	Type 3		542,662,510.48	41,474,779.05	893.65%		223.93	11.59	339.02%

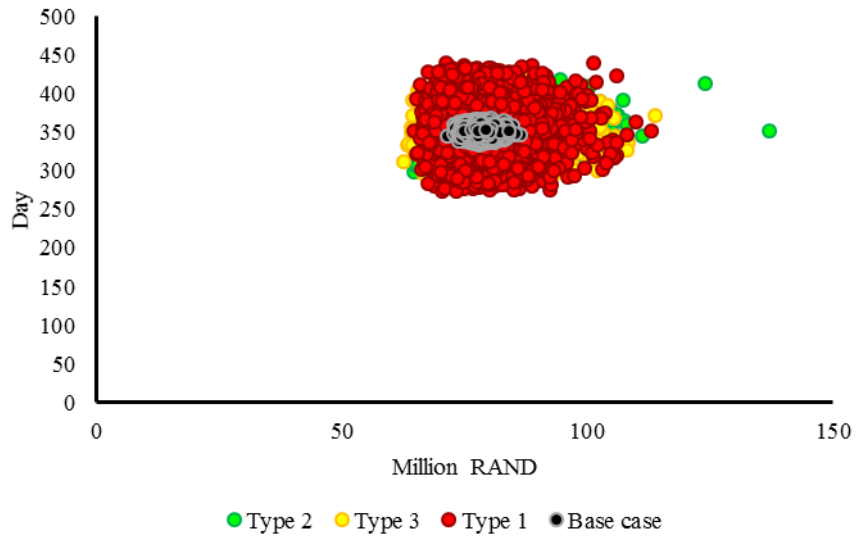
180 Furthermore, the cost and time of base case and correlation Type 1, 2 and 3 of  
 181 earthwork, bridge and pavement structures from Monte Carlo simulations overlaid in  
 182 scatterplots and are illustrated in Figures 1 to 3.

183 Cost correlations represent both threat and opportunity aspects. In fact, the  
 184 correlations cause the range of the total cost and time on both sides of the mean, which  
 185 is the threat of a total cost or time more substantial than the mean as well as the prospect  
 186 of a total cost or time smaller than the mean total cost or time as illustrated in three  
 187 overlaid scatterplots (Figs 1-3).



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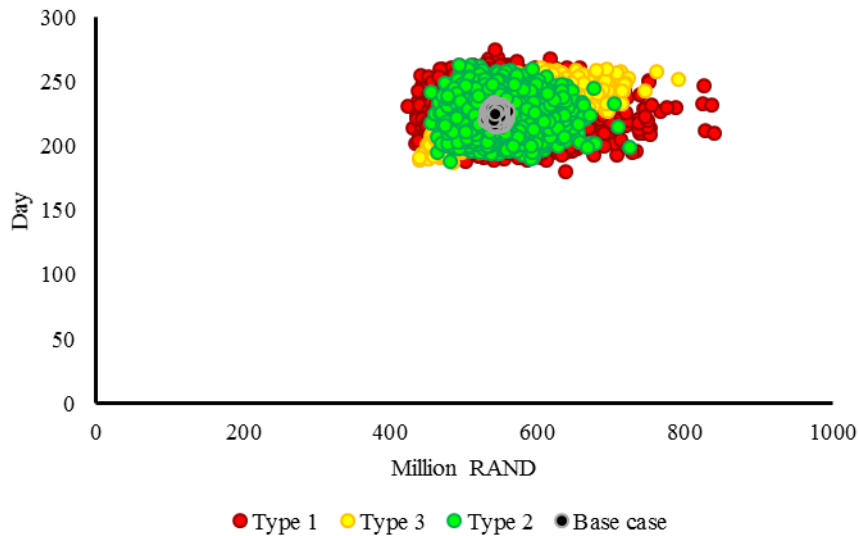
**Fig. 1.** Overlay scatterplot correlated cost and time of earthwork structure



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**Fig. 2.** Overlay scatterplot correlated cost and time of bridge structure



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**Fig. 3.** Overlay scatterplot correlated cost and time of pavement structure

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Based on the overlaying scatterplots of three types of correlation in the different structures of the highway project (Figs 1-3), and the results of means, standard deviations and variation of standard deviations of correlated cost and total time of the three structures of the highway project the following observations were made:

- The mean of the correlated cost and time are constant across different types of correlations (Table 1), which is confirmed by a constant centre of gravity in the cost and time-frequency plots (Figs. 1-3). This was expected because correlations affect the standard deviation and not the mean.
- The correlated means of total cost and total time are more significant than the deterministic estimated total cost and total time (Table 1) because the deterministic total cost and total time are the sums of the modes of costs and critical times of activities, but the correlated means of cost and total time are equal to the sum of the means of costs and critical times of activities; in other words, correlated cost and time are skewed to the right, (the mode is smaller than the mean) (Figs 1-3).
- The standard deviations of the cost and total time were increased considerably from base case (uncorrelated) to other correlation types, as shown in Table 3. The increases in total cost standard deviation and total time standard deviation are visible in the overlaying distribution cost and time plots, as well as scatterplots.
- Correlation Type 1 (correlation between repeated activity) caused the most substantial increase in the cost and time of standard deviation of earthwork and pavement structure, which increased by 1,104.35% and 756.055 in cost and time standard deviation of earthwork structure, respectively, and 1167.67% and 401.14% in cost and time of standard deviation of pavement structure,

218 respectively. On the other hand, correlation Type 2 (correlation between  
219 activities) caused the largest increase in the cost and time of standard deviation  
220 of bridge structure by 234.56% and 551.42%, respectively.

## 221 **6 Discussion and Conclusions**

222 This study examined the three types of correlation between the costs, times and  
223 cost/time of three main structures of highway projects and modelled their impacts on  
224 the distribution of the total cost and the total time of construction of highway structures.  
225 The effect of three types of correlations on the total cost and time standard deviation  
226 was modeled by Gaussian copula, a strong correlation model which allows correct  
227 generation of correlated variables, and the results were simulated by Monte Carlo  
228 simulation using actual data of 16.5 kilometers of a new highway constructed in South  
229 Africa.

230 The three structures of the case study have been organized in an arrangement that  
231 allows analysis of the effect of the three correlation types in terms of the magnitude of  
232 the correlation coefficients, types of the correlation matrix, marginal distributions, and  
233 some correlated activity costs. The dominant type of correlation on cost and time of  
234 different structures of the highway project was determined by comparing the  
235 differences in total cost and time variation of standard deviations. In the earthwork and  
236 pavement, correlation Type 1 causes the most substantial increase in total cost and time  
237 standard deviation, whereas, in the bridge structure, correlation Type 2 causes the most  
238 significant increase in total cost and time standard deviation.

239 Concerning the magnitude of the correlation coefficients, the results have clearly  
240 shown that the more significant the correlation coefficients ( $\rho=0.8$  in the pavement),  
241 the larger the increase on the total cost standard deviations (1104.35%), which is  
242 aligned with the findings of Moret and Einstein [4] study. However, in the time variable,  
243 the combination of the magnitude of the correlation coefficients and arrangement of  
244 critical activities on the network activity model of the structure determined the largest  
245 increase in the standard deviations of total time, which, in this study, is earthwork  
246 because  $\rho=0.8$  and all earthwork activities are located on a critical path (sequential  
247 network model without any dummy). The reason for this difference in type of  
248 correlation and considerable difference in the variation of standard deviation can be  
249 explained with the number of correlated costs and times: correlation Type 1 consists of  
250 the correlation between many activities' costs and times, while correlation Type 2  
251 consists of the correlation between few activities' costs and times in bridge structure.

252 The conclusion is that, as more costs and times are correlated with larger correlation  
253 coefficients, the effect on the total cost and time standard deviation becomes more  
254 substantial, which finding is supported by the studies of Bakhshi and Touran [2], Moret  
255 and Einstein [4]. The results of modelling three types of correlations on the cost and  
256 time of a highway project proved that the range of the total cost and total time increase  
257 considerably due to correlations. This study has shown that, although estimating the  
258 effect of correlations on cost is more straightforward, estimating the effect of  
259 correlations on time is more complex. Furthermore, the study proved that the



260 correlation impacts depend on the magnitude of the correlation coefficient, the  
261 correlation matrix, and the type of correlation. Also, the impact of the last is tightly  
262 connected to the number of activity costs and times that are correlated and the network  
263 model of the structure.

264 By considering the results of modelling correlation on the cost and time distribution  
265 of a highway project, it is now possible to determine the effect of such correlation in  
266 cost and time prediction models. Modelling correlation is an initial step towards a more  
267 comprehensive modelling of uncertainties of construction cost and time. The practical  
268 application of the proposed methodology can be extended to any construction project  
269 characterized by numerous repetitions of the same activities, particularly to the linear  
270 construction projects.

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